Knowing What You Don't Know: Nuclear Reactions, Effective Field Theory & Uncertainty Quantification

Daniel Phillips

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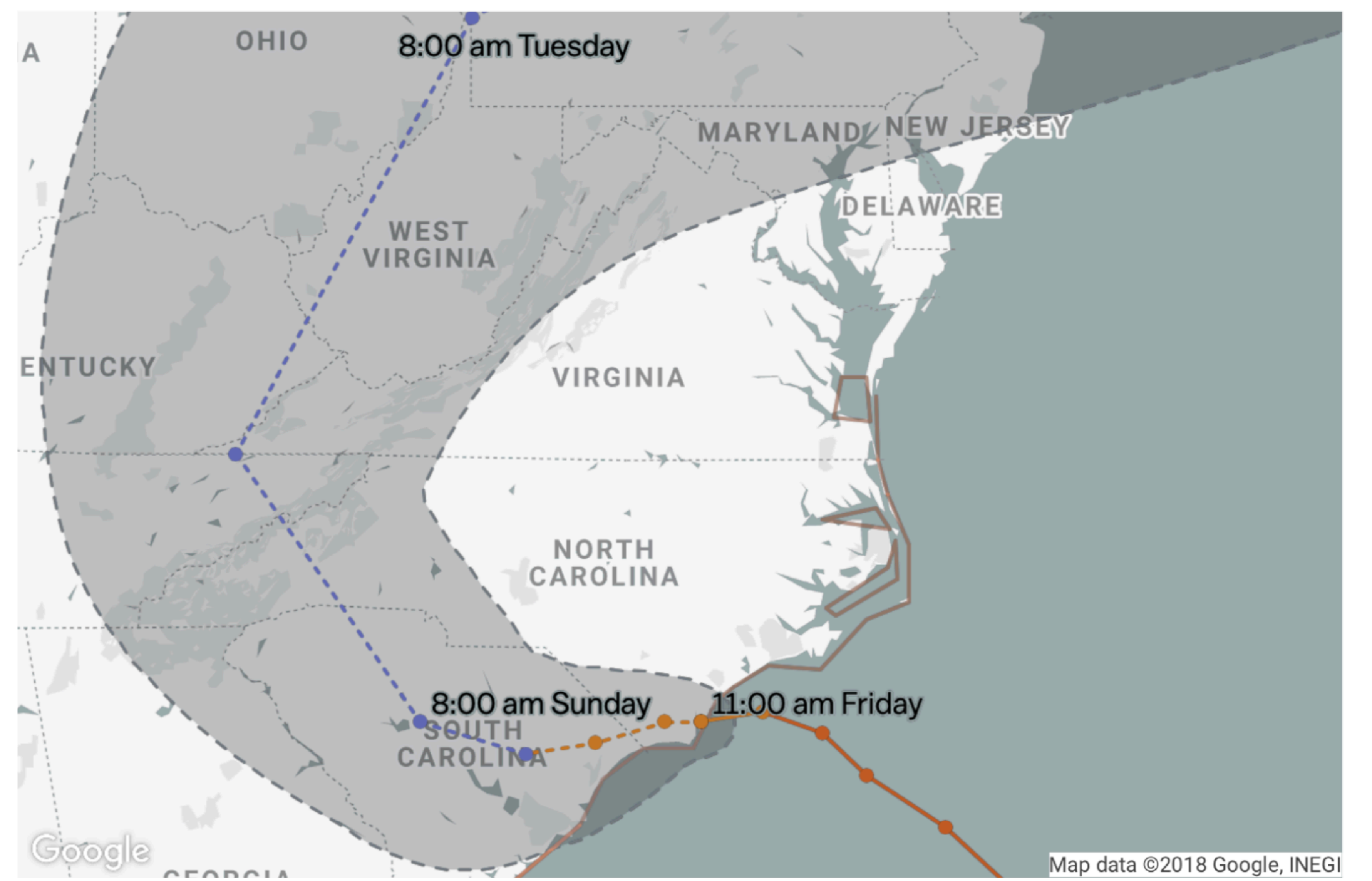
TU Darmstadt

EMMI

Research supported by the US DOE and by EMMI

Hurricane forecasting



Data: **National Hurricane Center**, NOAA. Updated: Sep 14, 2018, 2:00 pm

http://www.vox.com

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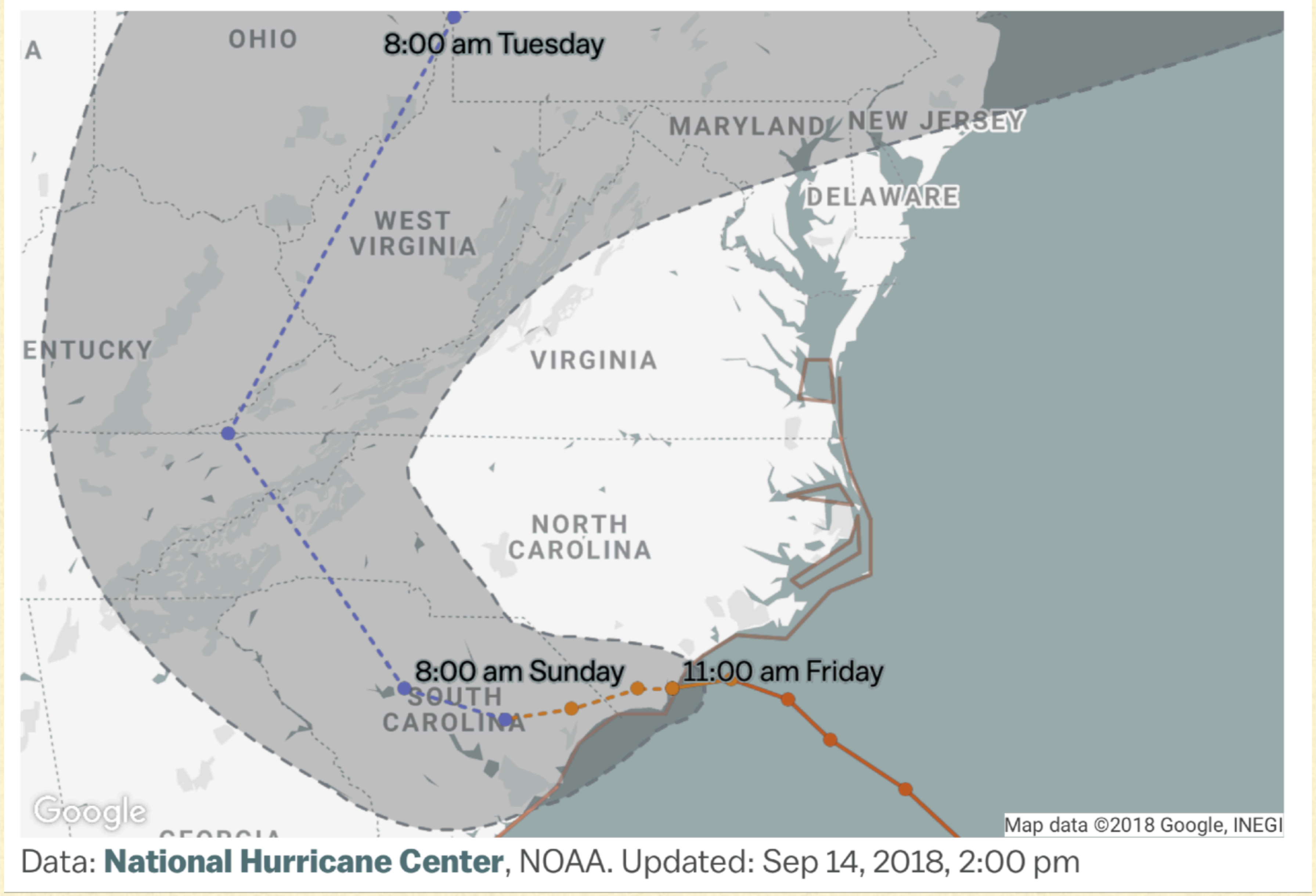

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- Parameterizations

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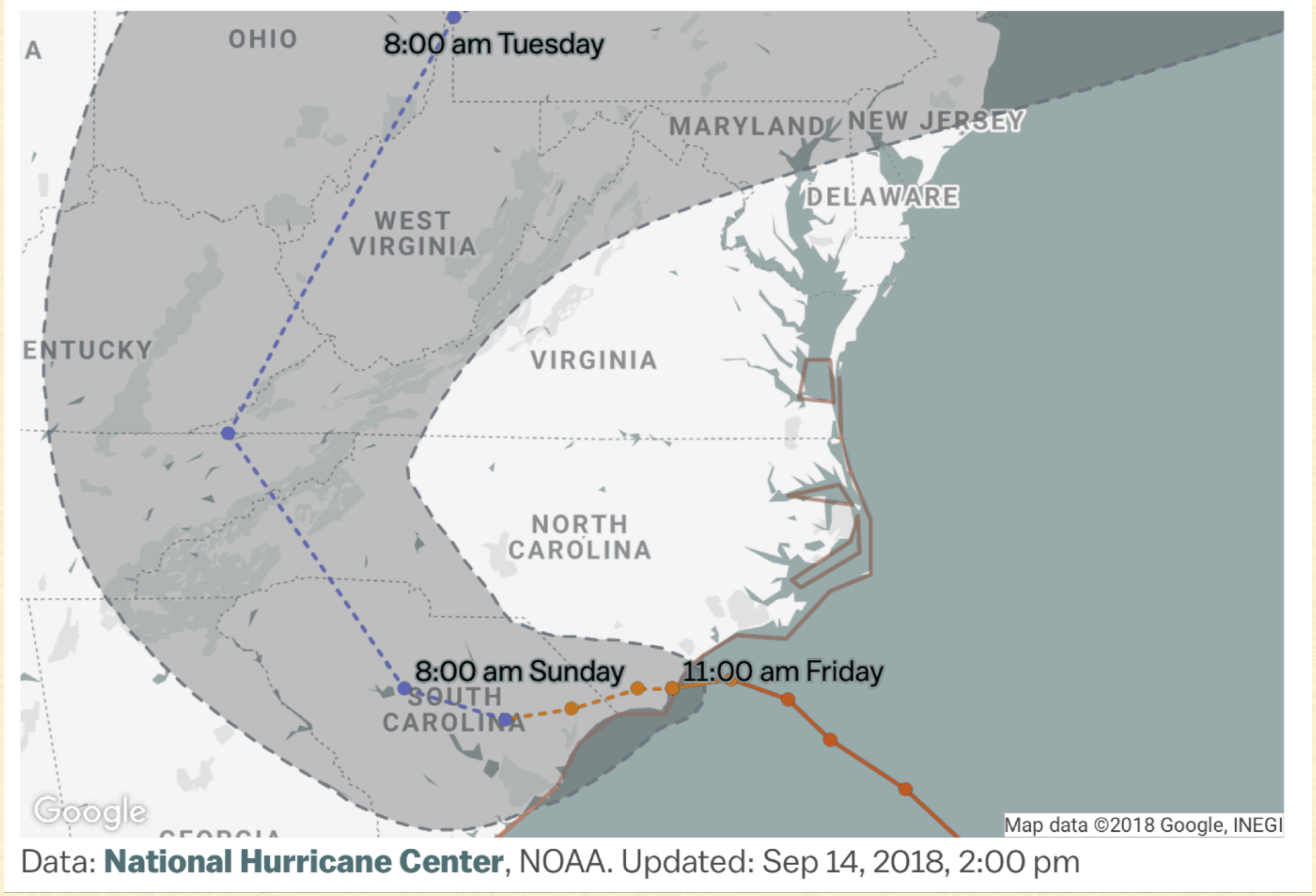

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- Evolve state forward in time (more computing!)
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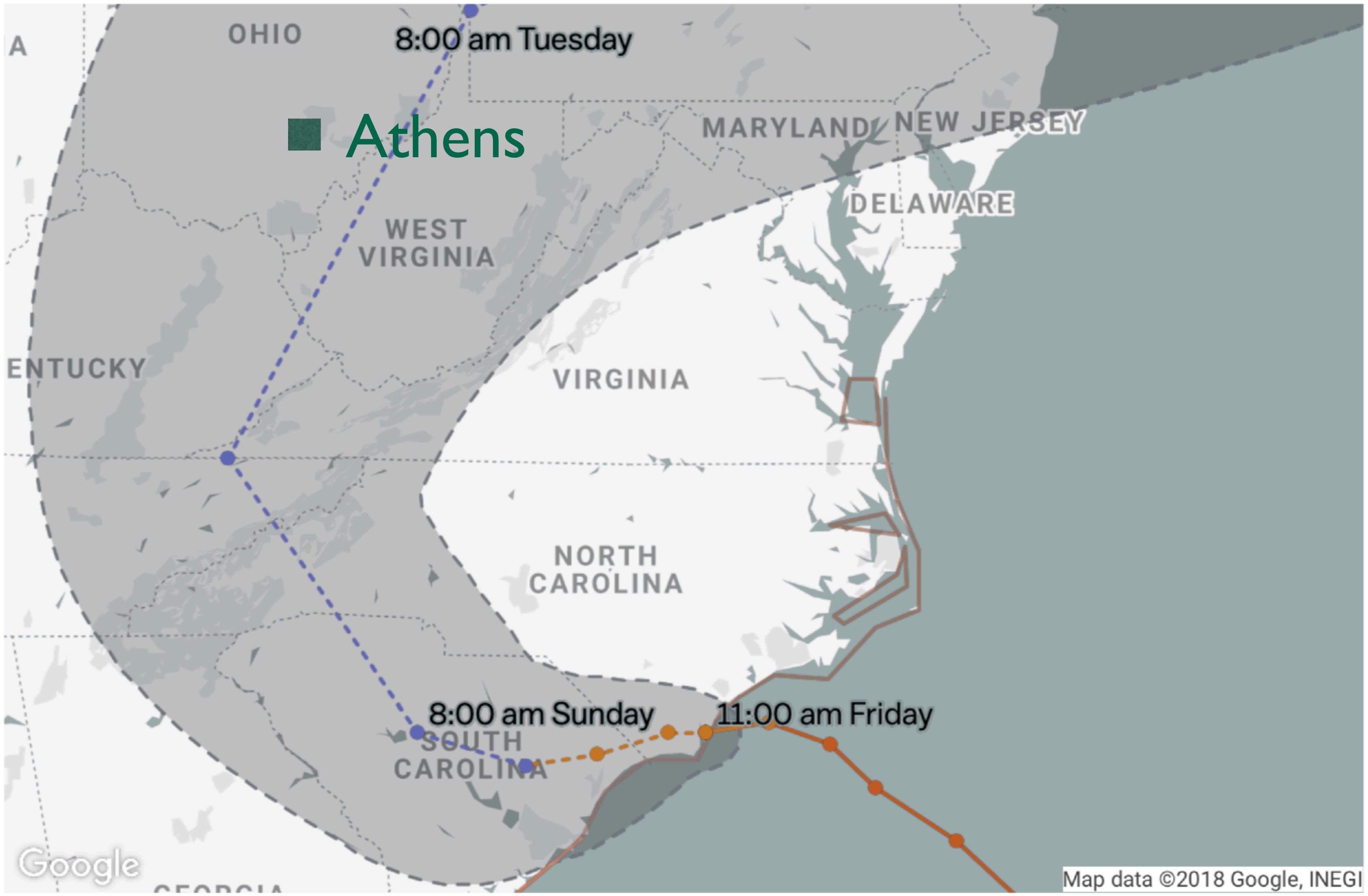

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- Forces: electromagnetic, strong nuclear
- Conservation laws, e.g., probability, energy, momentum
- Some parameterizations
- Accurate knowledge of initial state (nuclear structure)
- Computing to evolve state forward in time
- Uncertainty quantification

Outline

- What we do and don't know about the strong nuclear force
- EFT: organizing what we know, constraining what we don't
- EFT truncation errors from a Bayesian analysis: NN scattering
- EFT for halo nuclei: universal formula for $\gamma + {}^{A}Z \rightarrow {}^{A-1}Z + n$
- Uncertainty quantification for fusion: ^{7}Be(p,γ) at solar energies
- Conclusion

Potentials from particle exchange

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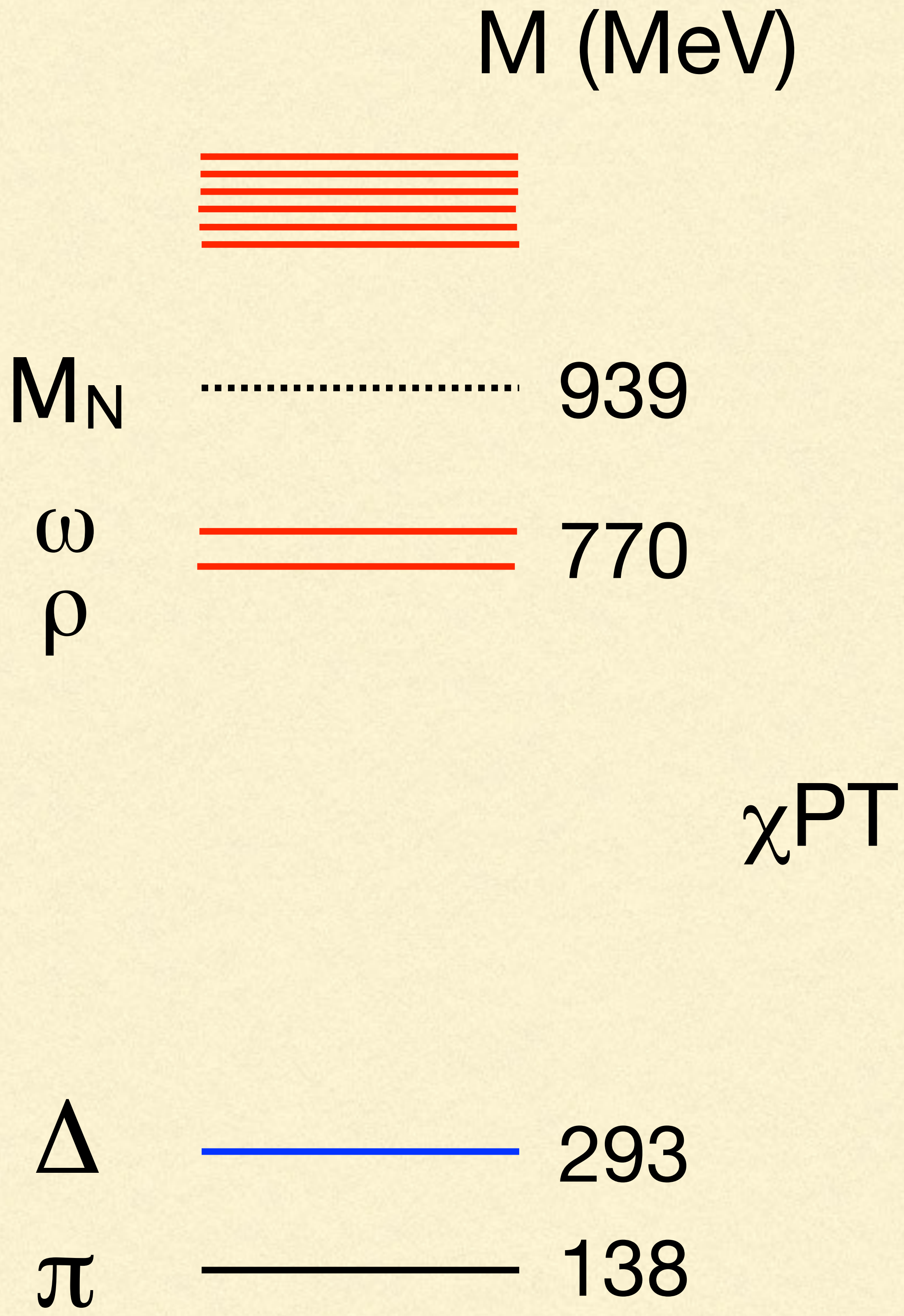
- Longest range forces generated by lightest particles

The long and the short of hadron physics



M (MeV)

M_N 939

ω ρ 770

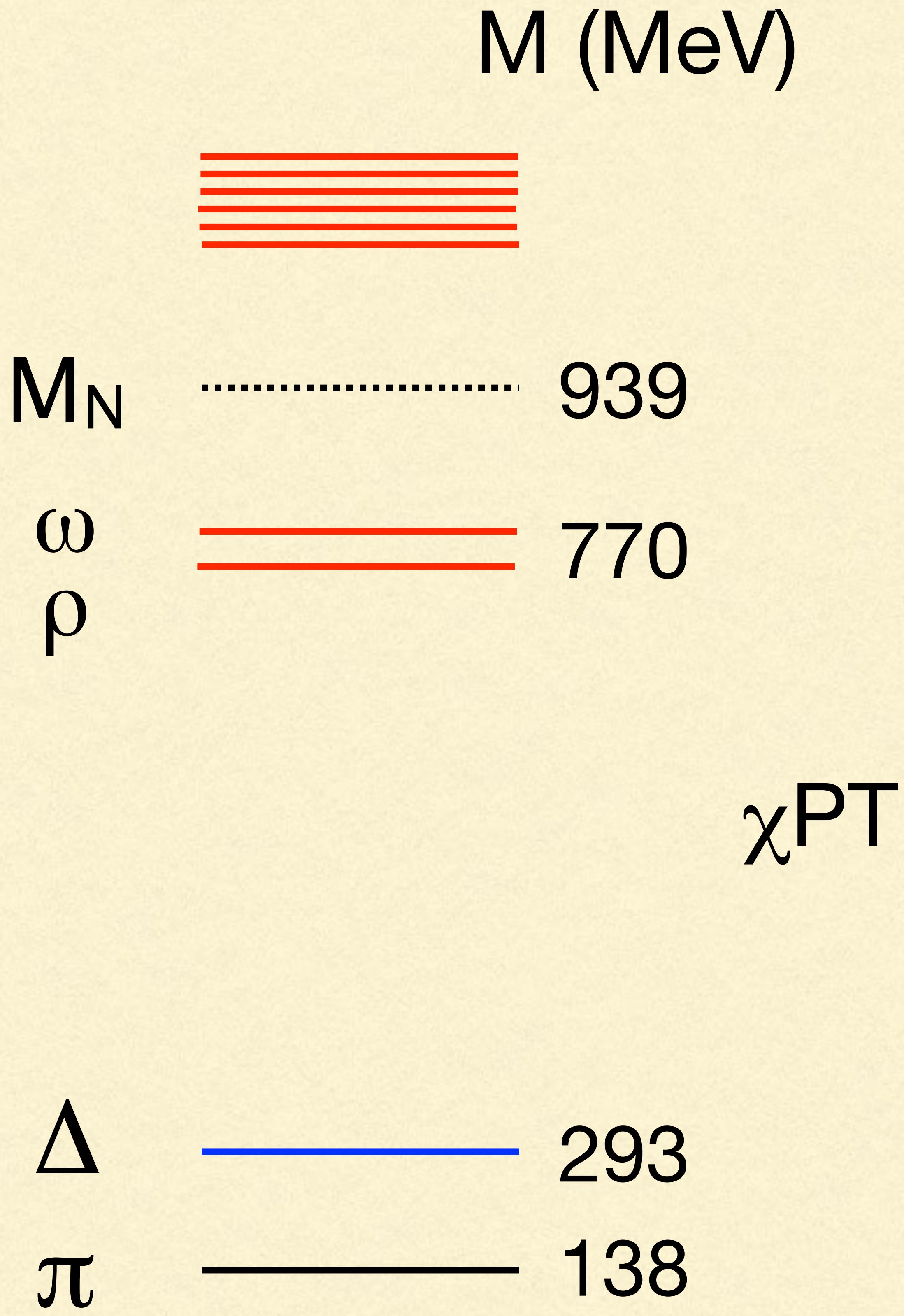
χPT

Δ 293

π 138

The long and the short of hadron physics

- Spectrum of QCD bound states



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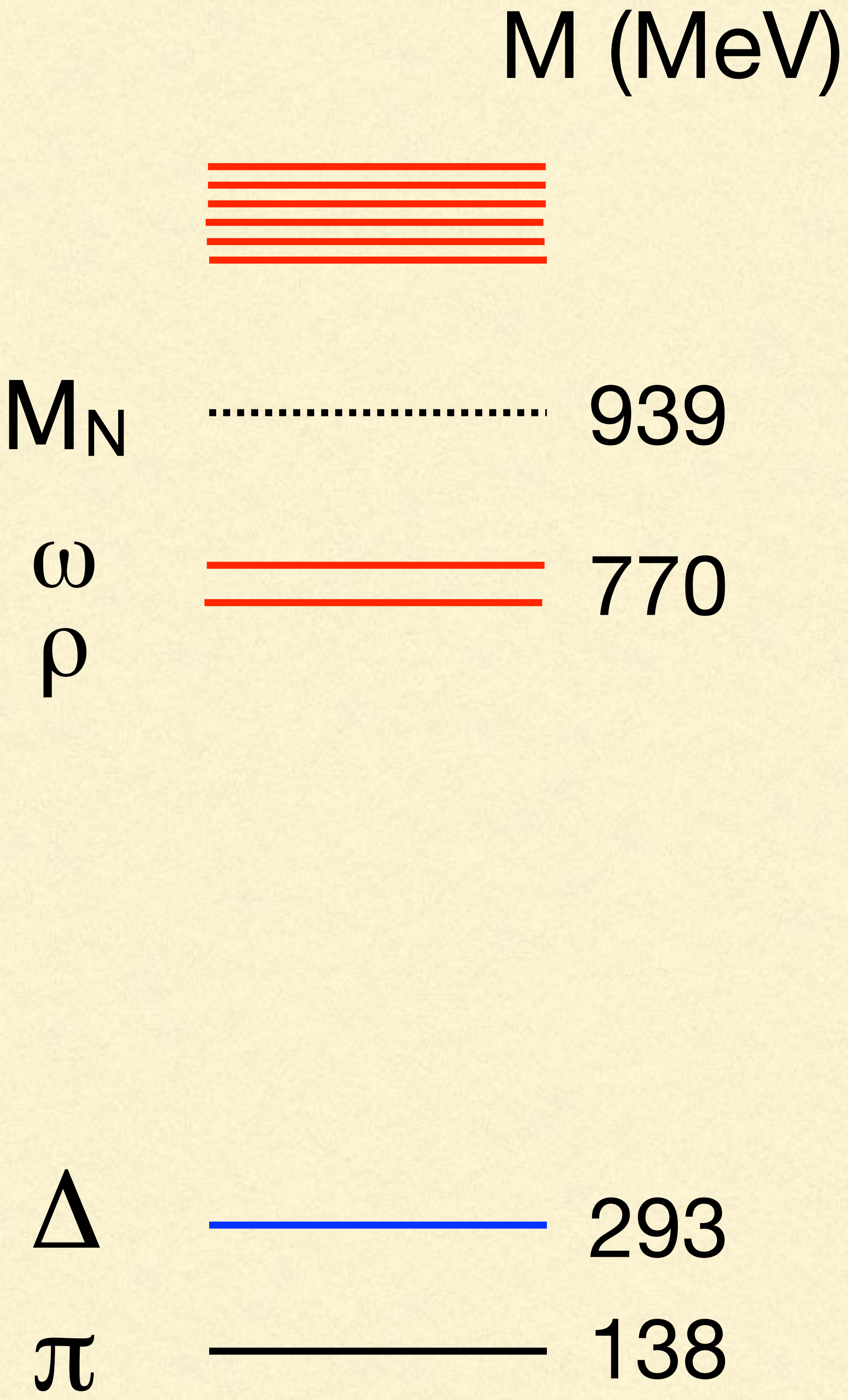

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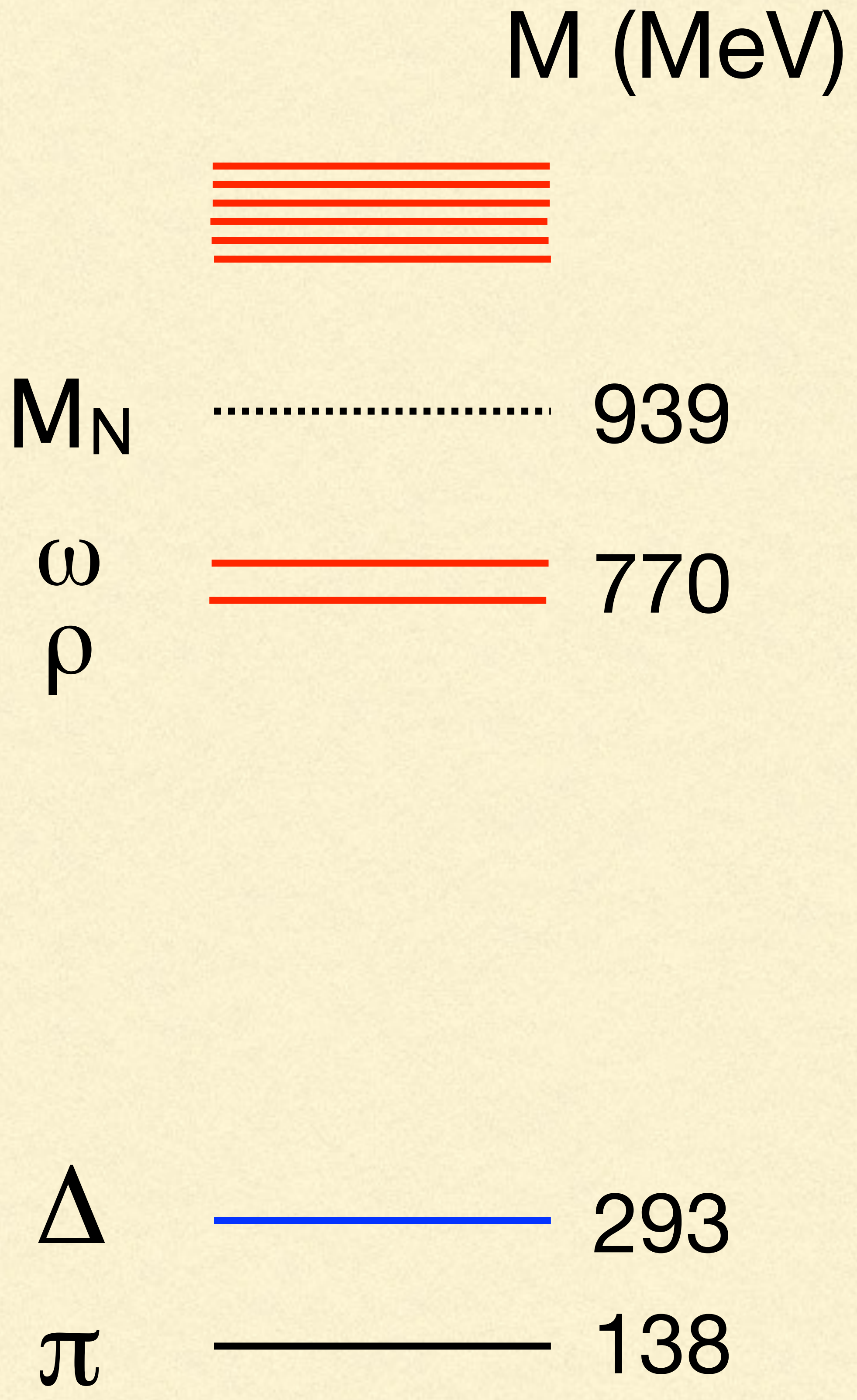
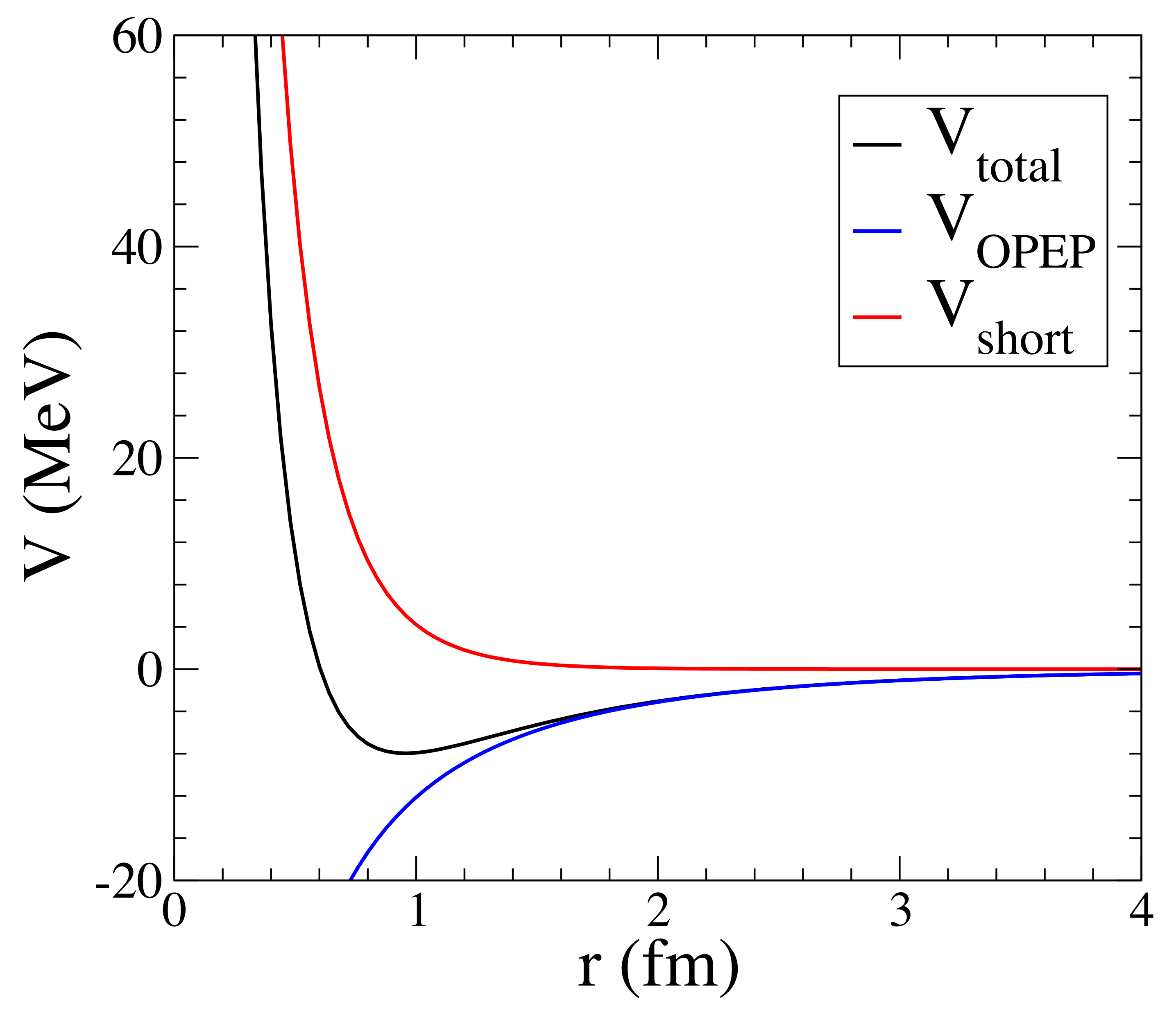
- For probe energies ~a hundred MeV, simplifications of the rich QCD dynamics emerge: processes dominated by πs (and Δs)
- Pion exchange generates longest-range part of NN force
- But short-distance dynamics too



The NN potential: a cartoon



- Long-range part generated by one-pion exchange
- Intermediate ranges: multiple pion exchange
- Short ranges: "other stuff" exchange
- Needs to be parameterized, then fit to NN scattering data

Effective Field Theory

- Simpler theory that reproduces results of full theory at long distances
- Short-distance details irrelevant for long-distance (low-momentum) physics, e.g. multipole expansion
- Expansion in ratio of physical scales: $p/\Lambda_b=\lambda_b/r$
- Symmetries of underlying theory limit possibilities: all possible terms up to a given order present in EFT
- Short distances: unknown coefficients at a given order in the expansion need to be determined. Symmetry relates their impact on different processes
- Examples: standard model, chiral perturbation theory, Halo EFT

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Error grows as first omitted term in expansion

χEFT for nuclear forces

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  Weinberg (1990), apply χPT to V, i.e. expand it in P=p/Λ_b

$$(E - H_0)|\psi\rangle = V|\psi\rangle$$

$$V = V^{(0)} + V^{(2)} + V^{(3)} + \dots$$

Ordonez, Ray, van Kolck (1996); Epelbaum, Meissner, Gloeckle (1999); Entem, Machleidt (2001)

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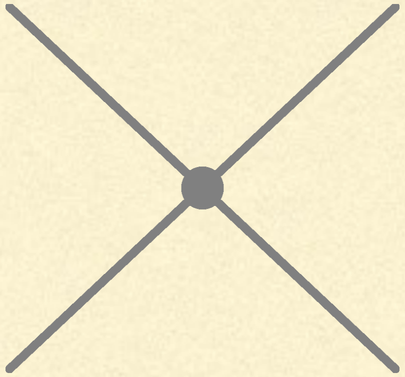
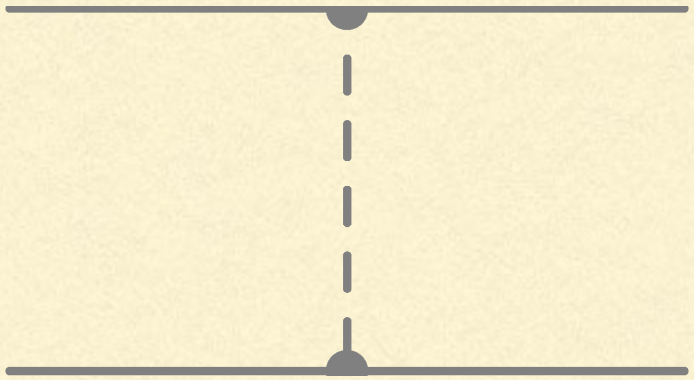
- Leading-order V:

$V^{(0)} =$



$+$



;

$$\langle \mathbf{p}'|V|\mathbf{p}\rangle = C^{3S1}P_{3S1} + C^{1S0}P_{1S0} + V_{1\pi}(\mathbf{p}' - \mathbf{p})$$

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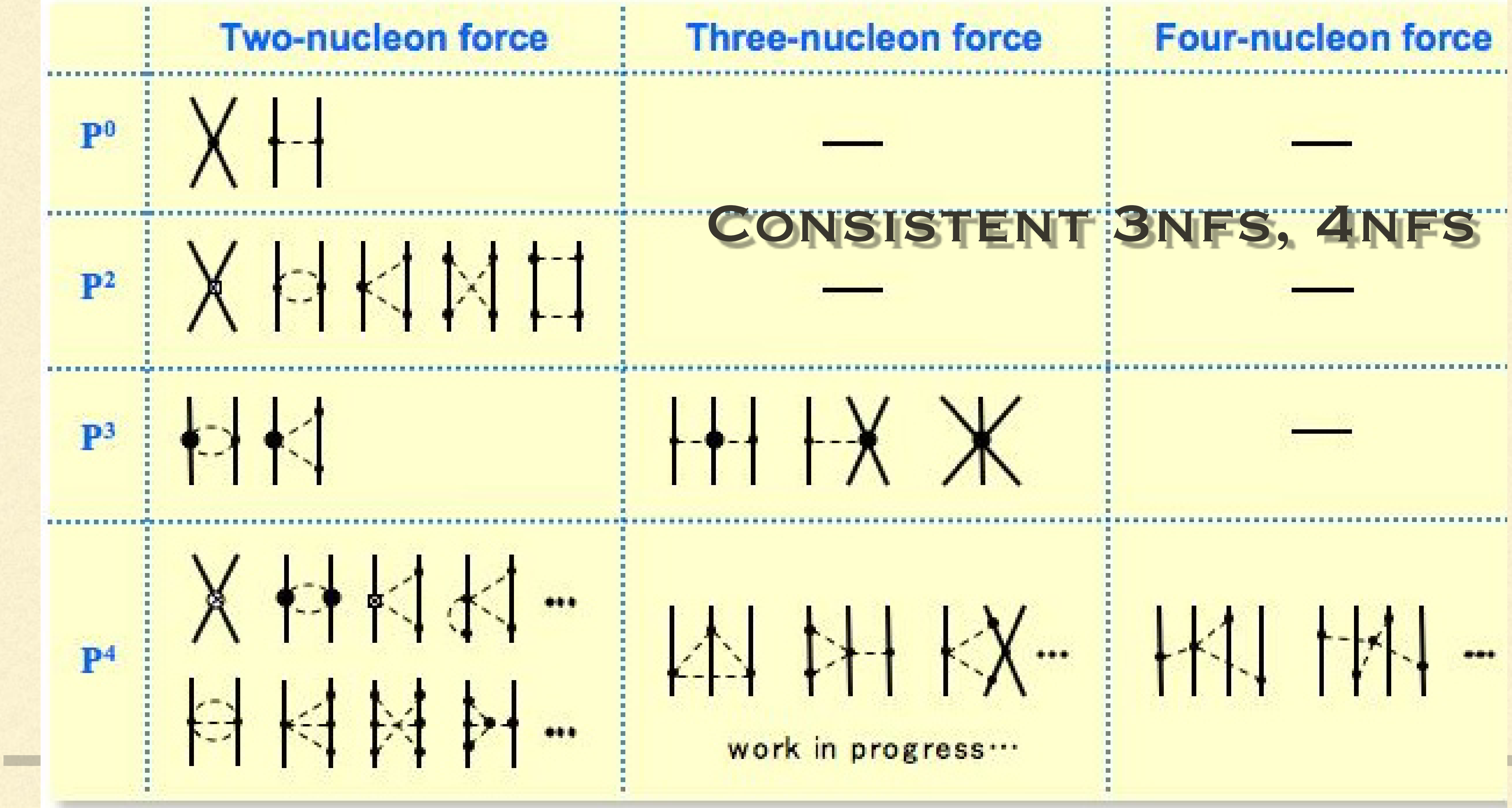

	Two-nucleon force	Three-nucleon force	Four-nucleon force
P⁰		—	—
P²		—	—
P³			—
P⁴	··· ···	··· work in progress···	···

CONSISTENT 3NFS, 4NFS

Figure courtesy E. Epelbaum

NN scattering

Epelbaum, Krebs, Meissner, PRL (2015); EPJA (2015)

- Potential regulated by local function, parameterized by R

$$\sigma_{np}(E_{\text{lab}}) = \sigma_{\text{LO}} \sum_{n=0}^{k} c_n(p_{\text{rel}}) \left(\frac{p_{\text{rel}}}{\Lambda_b} \right)^n$$

EKM state
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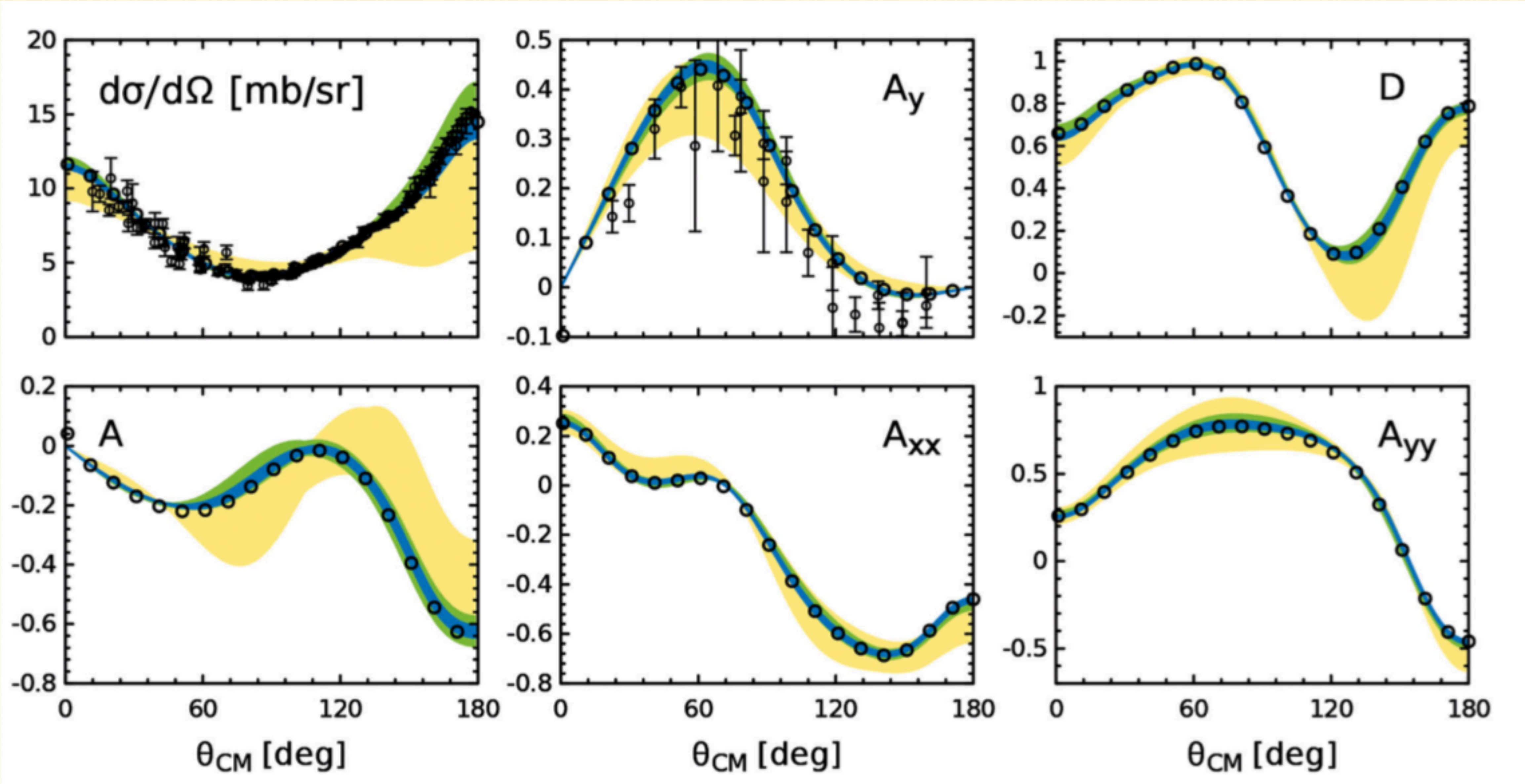

R=0.9 fm
here

np observables at Elab=96 MeV at NLO, N^{2}LO, N^{3}LO (k=2, 3, 4)

Successes in A=3-12

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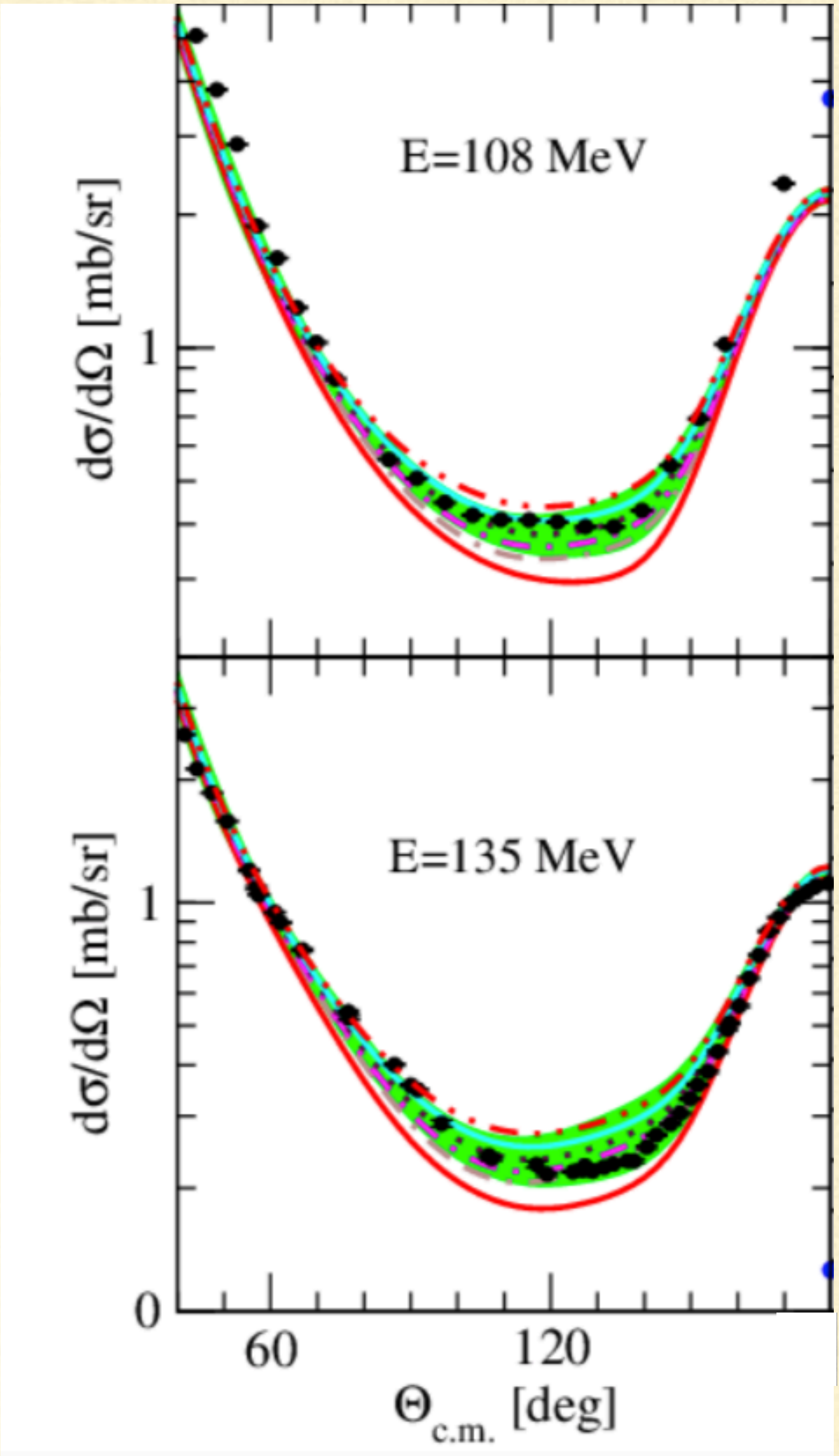

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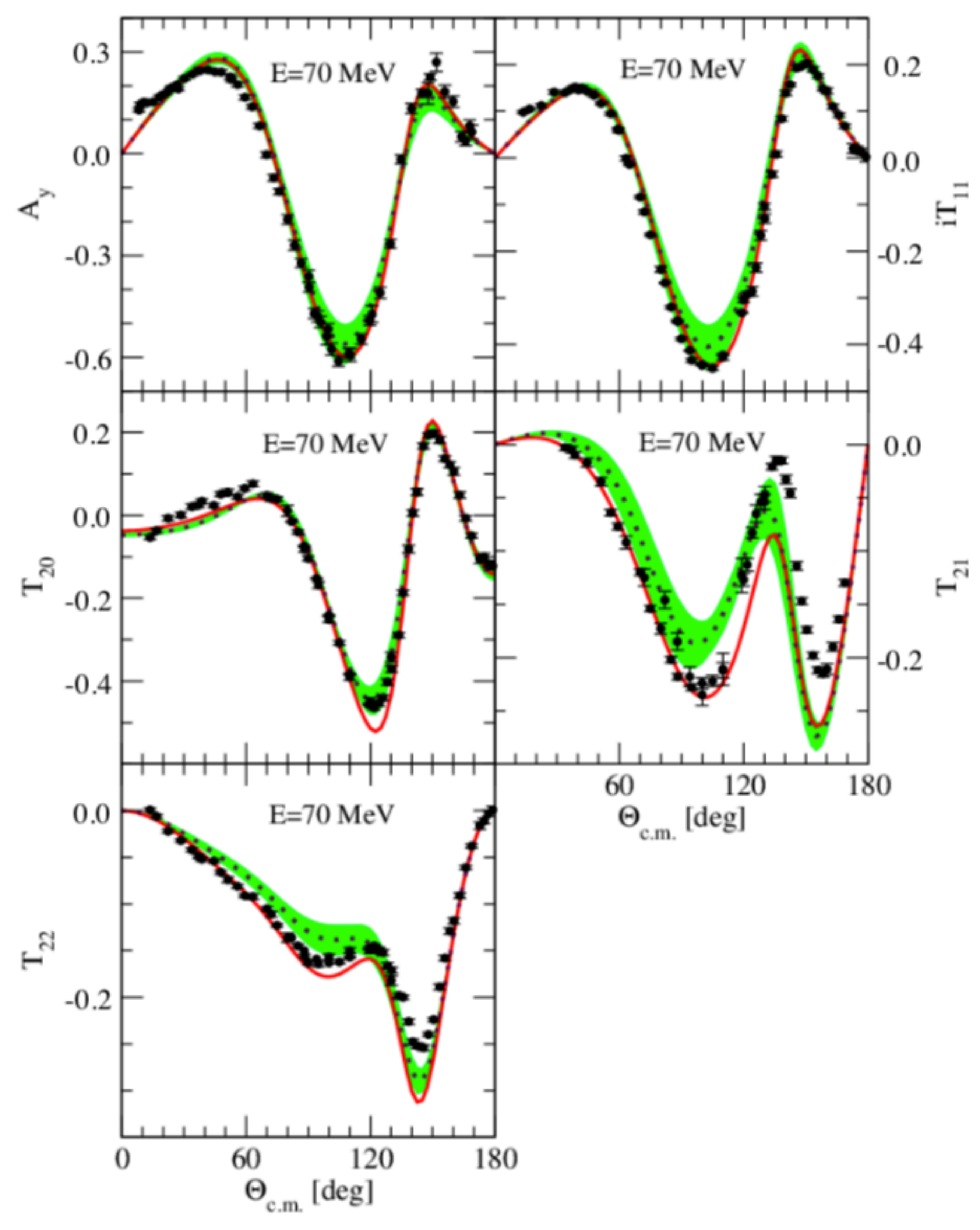

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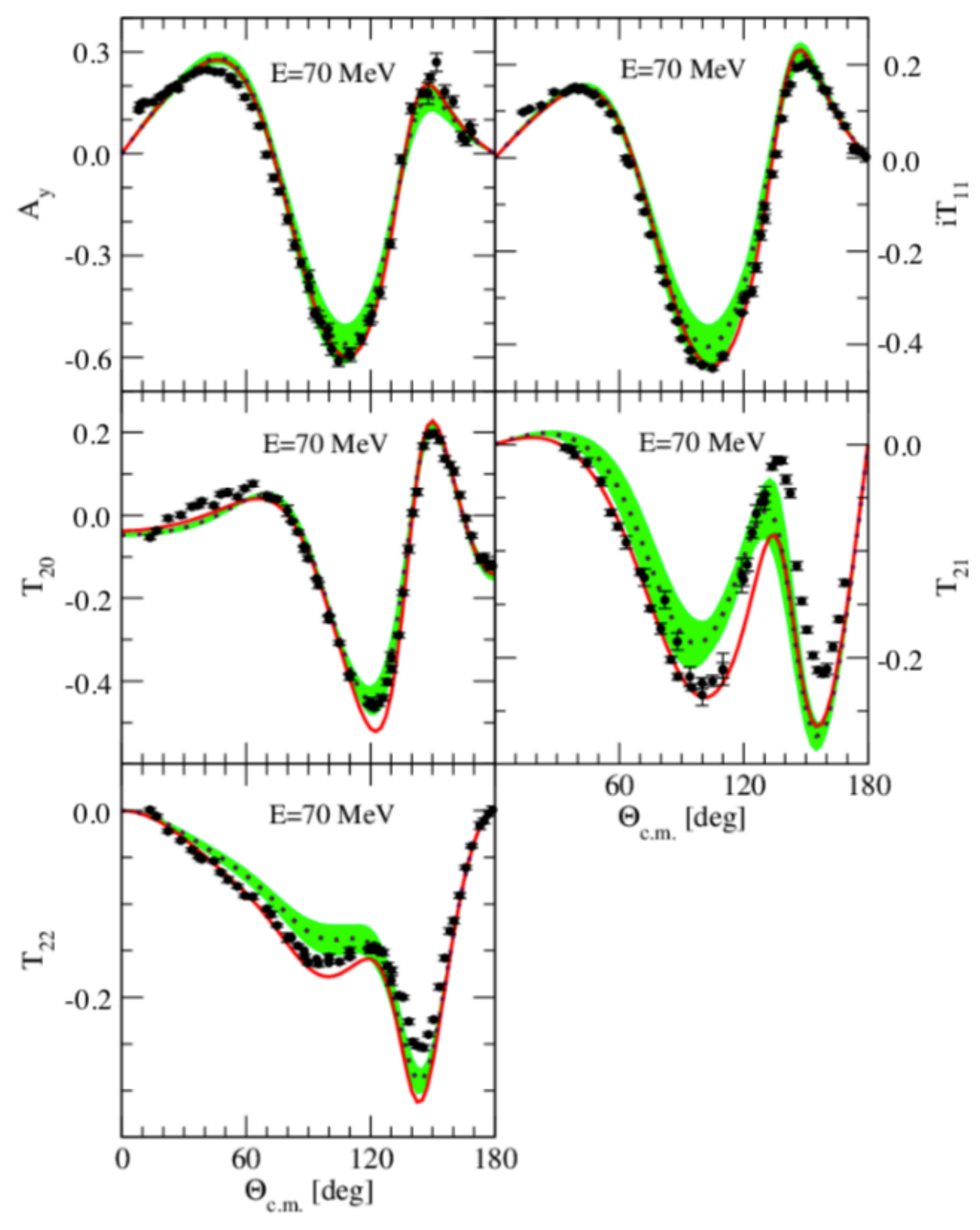

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- χEFT at N^2LO reproduces binding energies of light nuclei reasonably well

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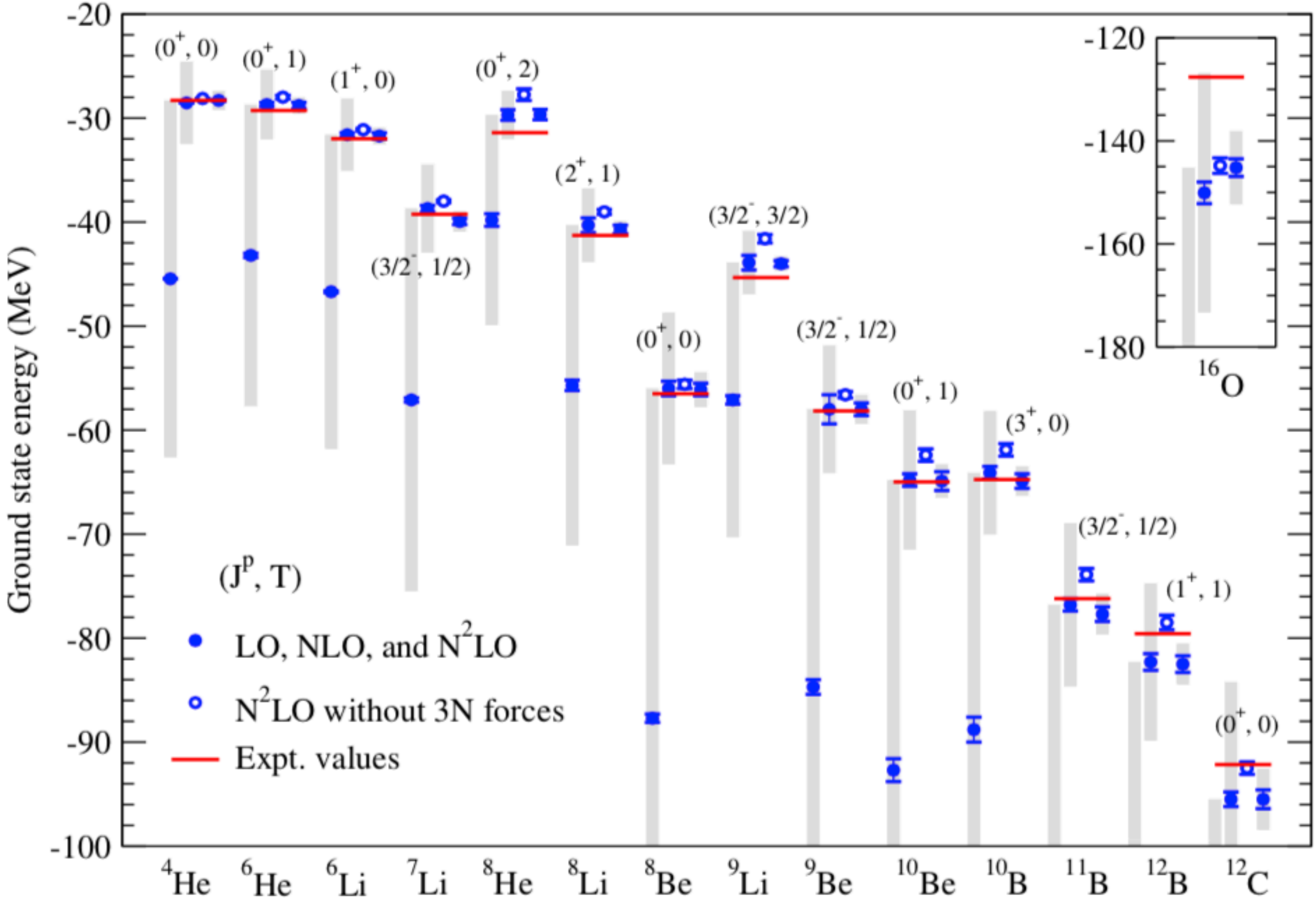

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  - In fact $\{c_n\}=\{1,-0.46,0.75\}$. What then is expectation for c_3?
- One possibility: $c_3=\max\{c_0,c_1,c_2\}$

Updating

Epelbaum, Krebs, Meissner (2014)
cf. McGovern, Griesshammer, Phillips (2013); many others.

Bayesian tools

Thomas Bayes (1701?-1761)

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$$\mathrm{pr}(A|B,I) = \frac{\mathrm{pr}(B|A,I)\mathrm{pr}(A|I)}{\mathrm{pr}(B|I)}$$

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Posterior

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Probability as
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Marginalization: $\mathrm{pr}(x|\mathrm{data},I) = \int dy\, \mathrm{pr}(x,y|\mathrm{data},I)$

Allows us to integrate out "nuisance" (e.g. higher-order) parameters

Probability for EFT coefficients

Furnstahl, Klco, DP, Wesolowski, PRC,2015 after Cacciari and Houdeau, JHEP, 2011

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Result: $$\mathrm{pr}(c_{k+1}|c_0, c_1, \ldots, c_k) \propto \begin{cases} 1 & \text{if } c_{k+1} < c_{\max} \\ \left(\frac{c_{\max}}{c_{k+1}}\right)^{k+2} & \text{if } c_{k+1} > c_{\max} \end{cases}$$

$[-c_{max}X_0x^{k+1}, c_{max}X_0x^{k+1}]$ is a $\frac{k+1}{k+2} * 100\,\%$ DoB interval

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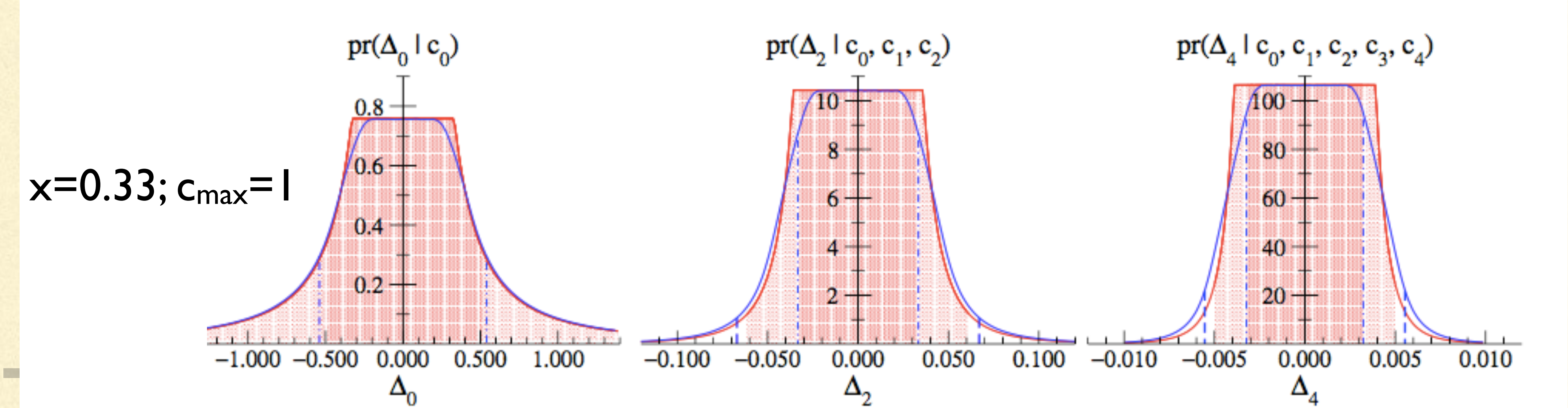

$x=0.33;\ c_{max}=1$

NN scattering cross sections

Epelbaum, Krebs, Meissner, EPJA, 2015

- NN cross section at T_{lab}=50, 96, 143, 200 MeV
- Potential regulated by local function, parameterized by R. Here: R=0.9 fm data
- Results at LO, NLO, N^2LO, N^3LO, N^4LO (k=0, 2, 3, 4, 5)

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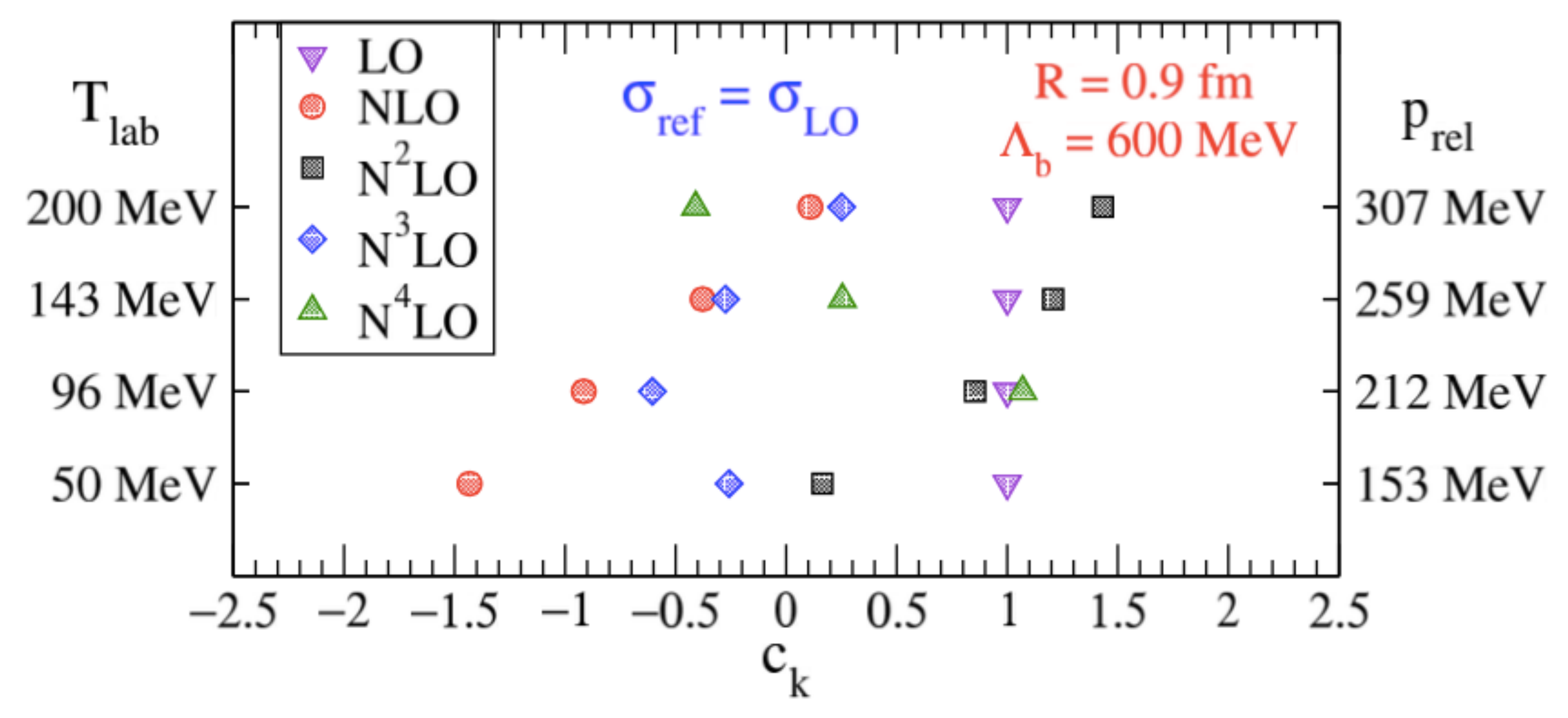
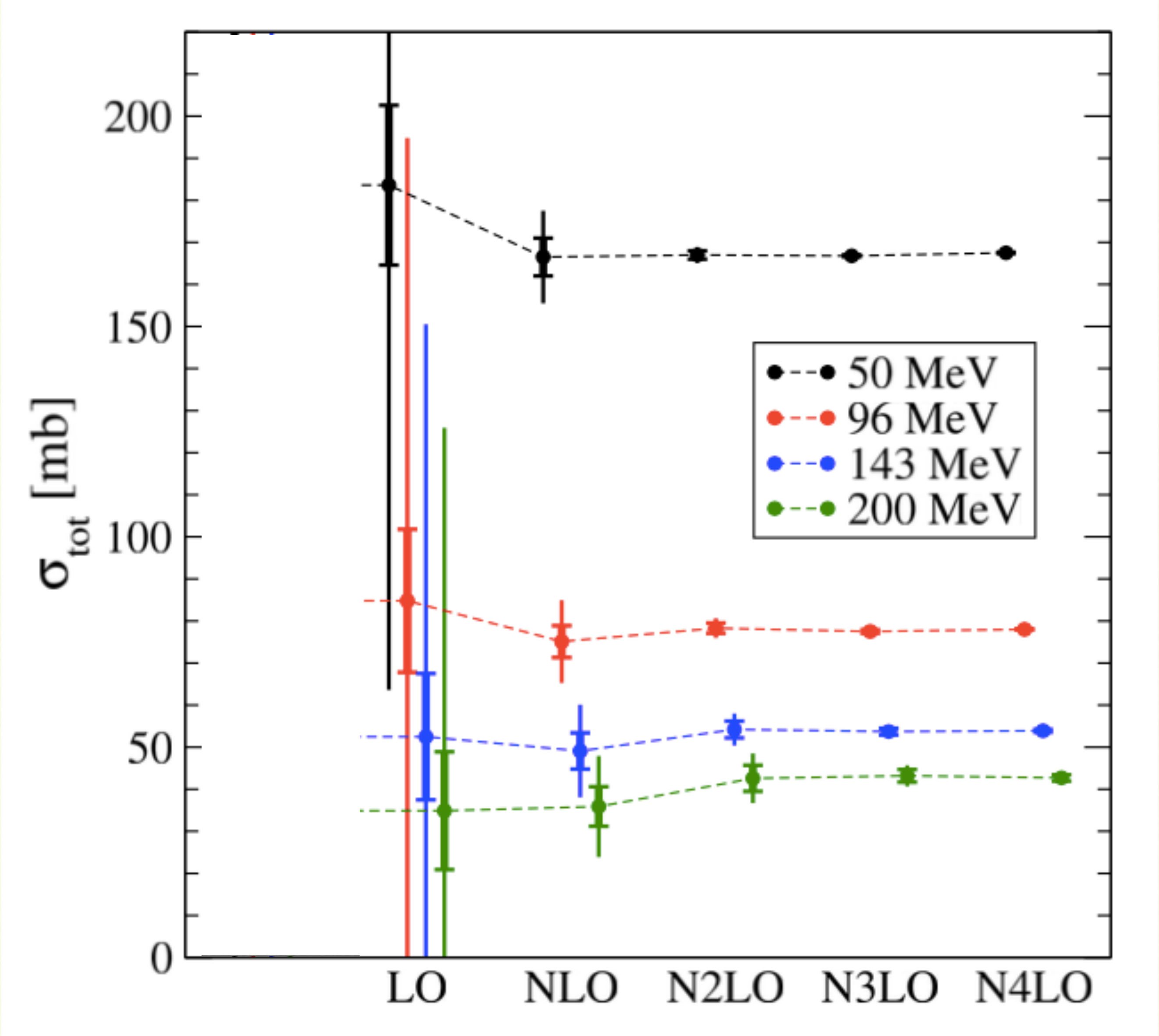
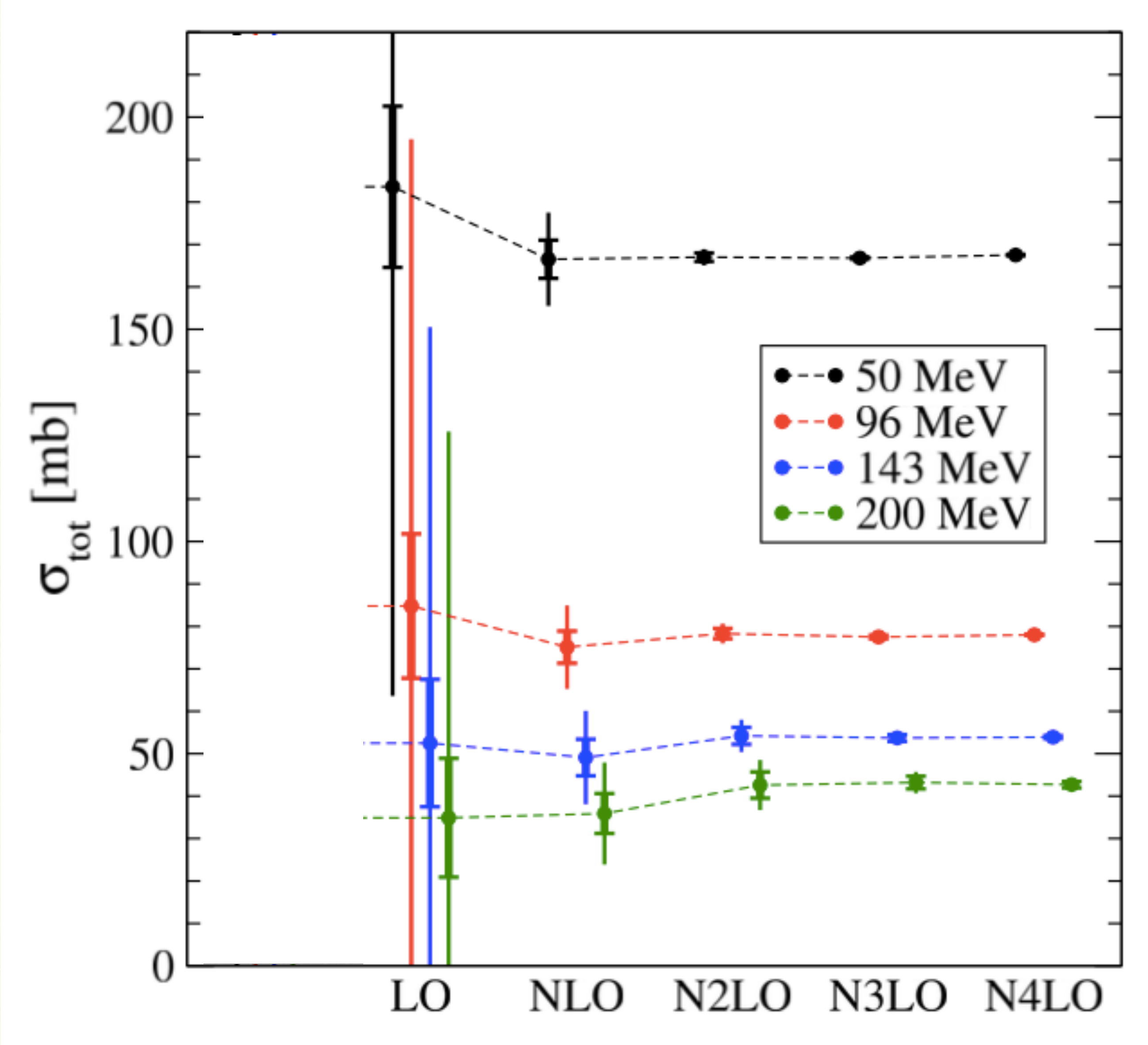


Results



σ_{tot} [mb]
0
50
100
150
200
LO
NLO
N2LO
N3LO
N4LO
50 MeV
96 MeV
143 MeV
200 MeV

Results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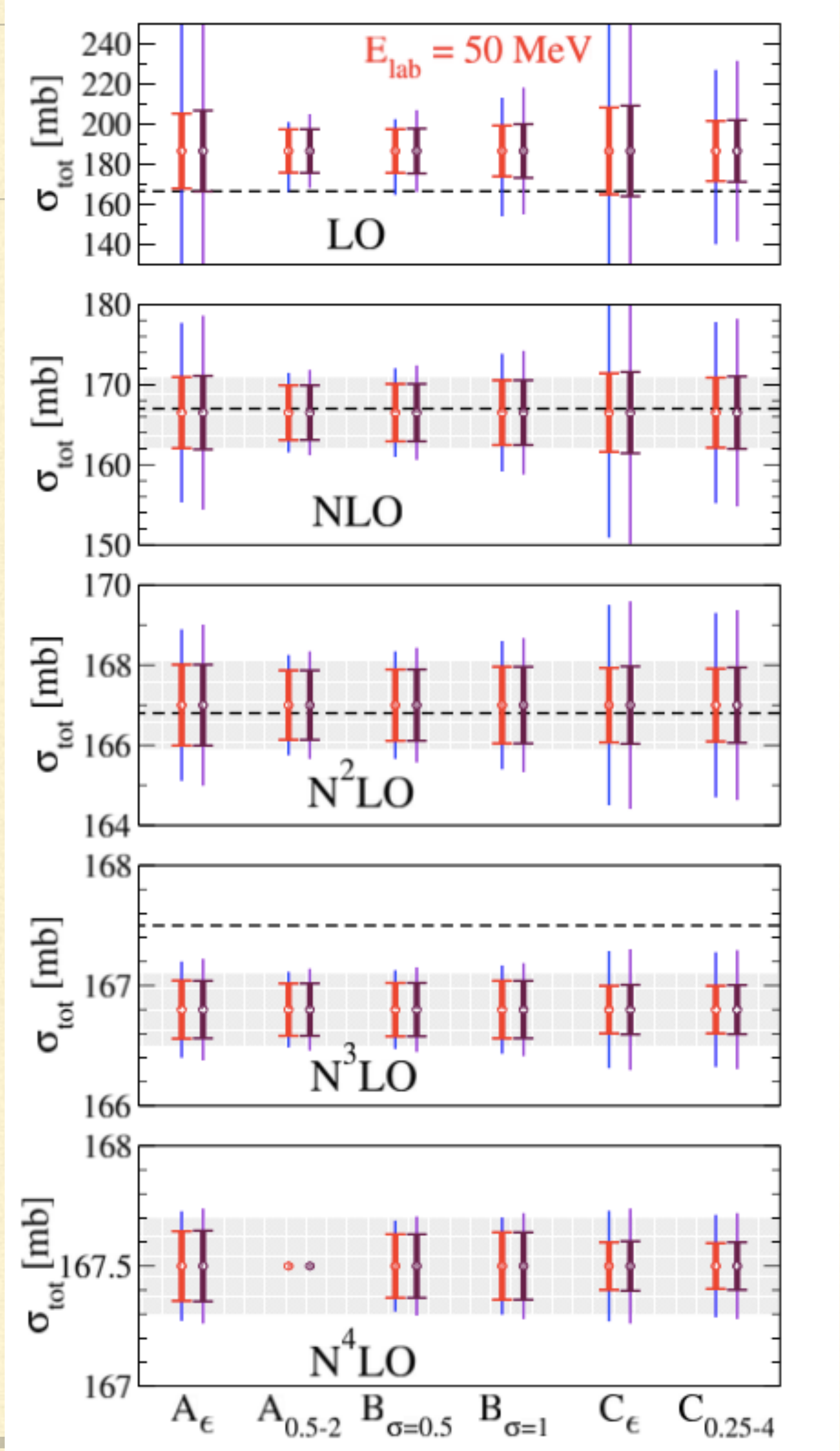
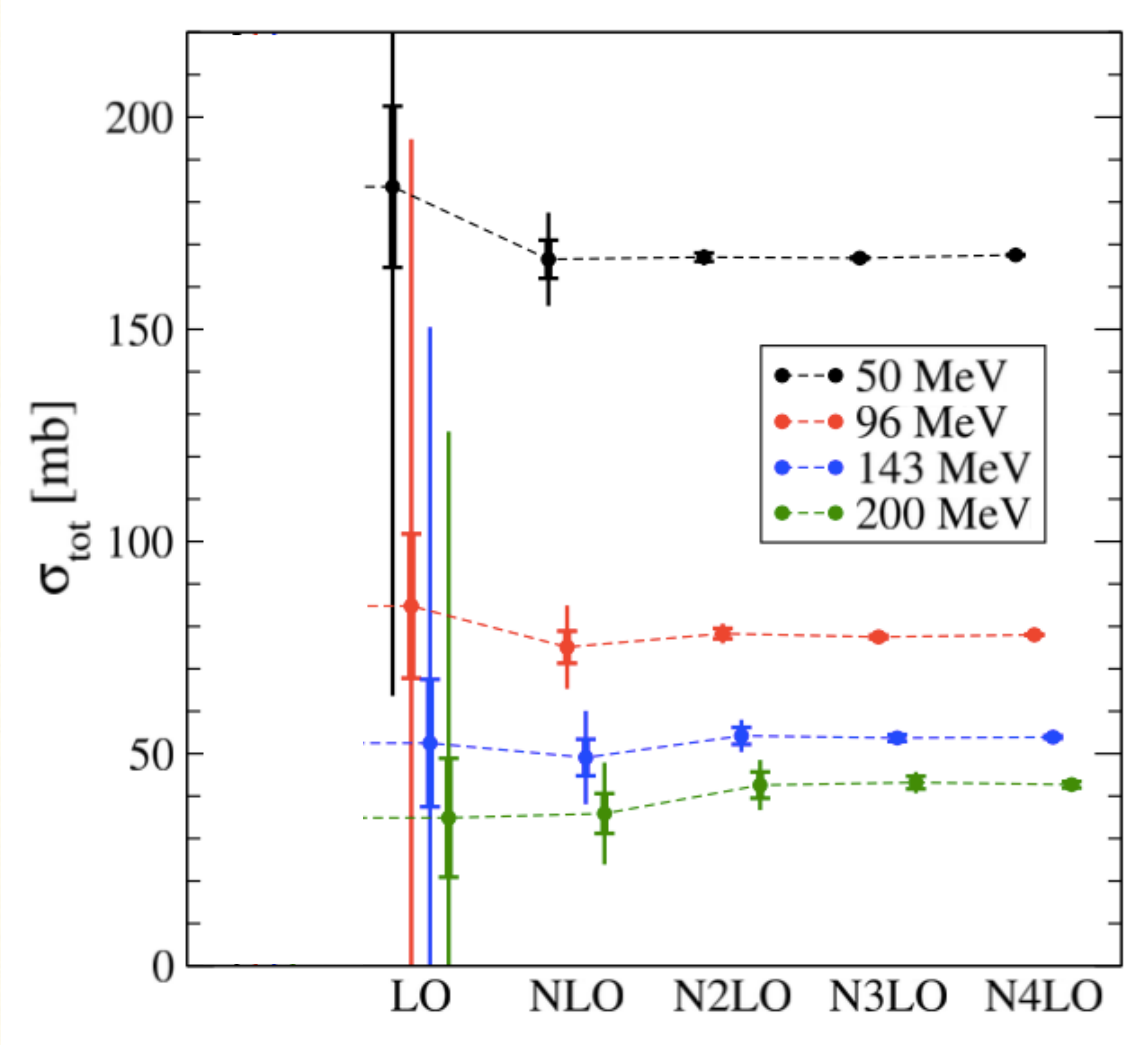
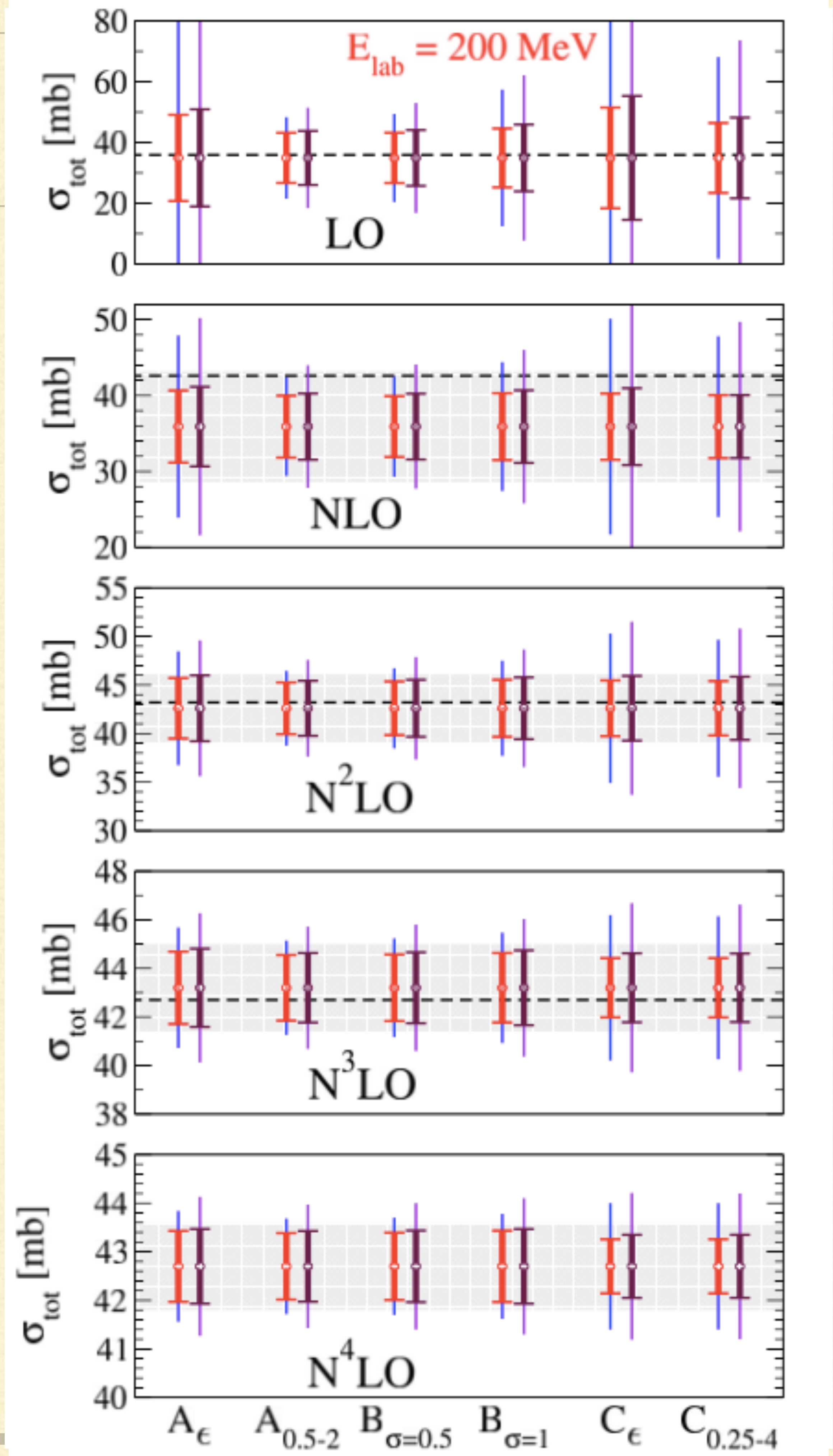

E_lab = 50 MeV
σ_tot [mb]
240
220
200
180
160
140
LO
σ_tot [mb]
180
170
160
150
NLO
σ_tot [mb]
170
168
166
164
N²LO
σ_tot [mb]
168
167
166
N³LO
σ_tot [mb]
168
167.5
167
N⁴LO
A_ε
A_0.5-2
B_σ=0.5
B_σ=1
C_ε
C_0.25-4

Results



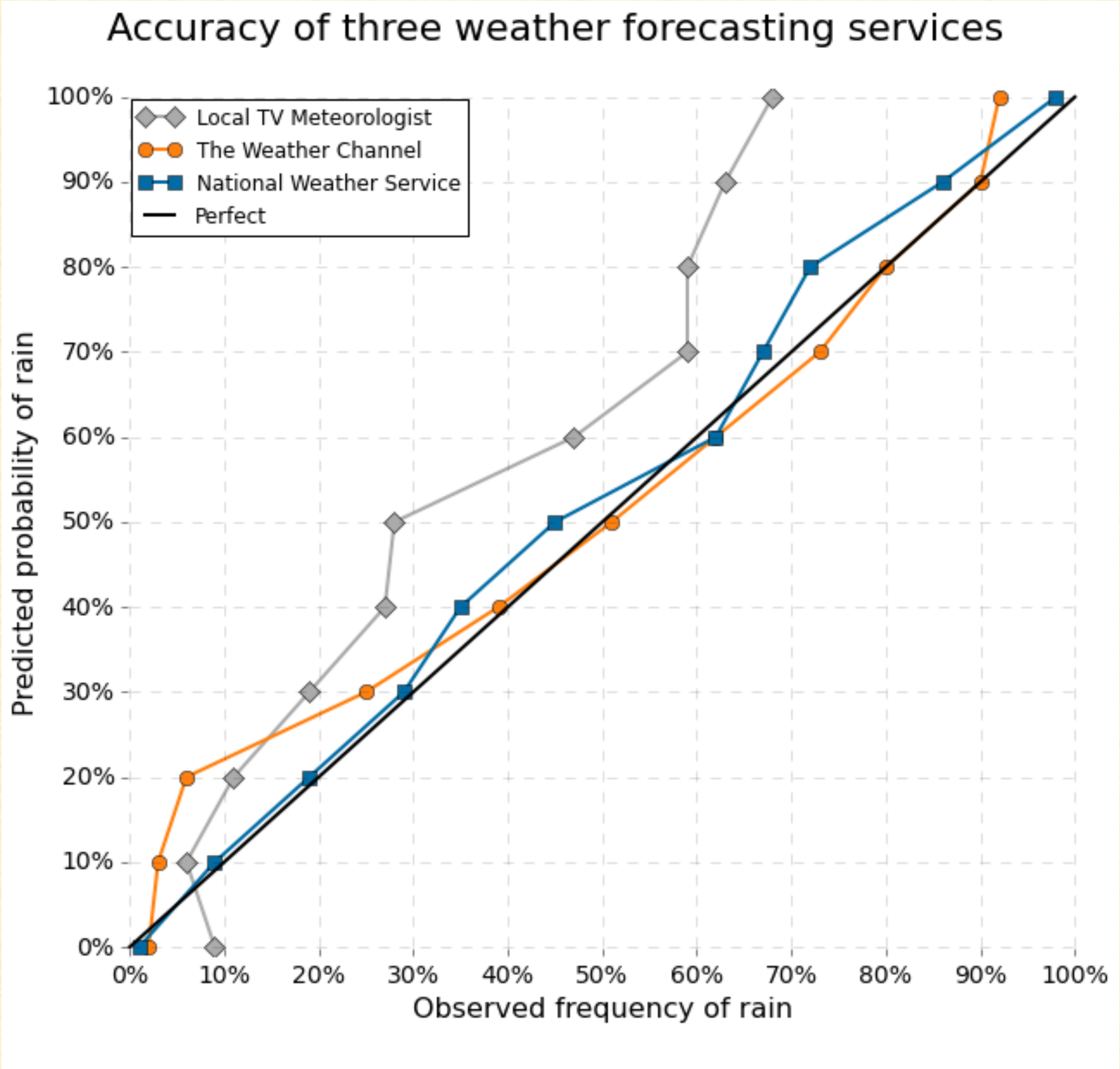
σ_tot [mb]
0
50
100
150
200
LO
NLO
N2LO
N3LO
N4LO
50 MeV
96 MeV
143 MeV
200 MeV



E_lab = 200 MeV
σ_tot [mb]
LO
0
20
40
60
80
NLO
20
30
40
50
N^2LO
30
35
40
45
50
55
N^3LO
38
40
42
44
46
48
N^4LO
40
41
42
43
44
45
A_ϵ
A_0.5-2
B_σ=0.5
B_σ=1
C_ϵ
C_0.25-4

The well-calibrated EFTist

The well-calibrated EFTist

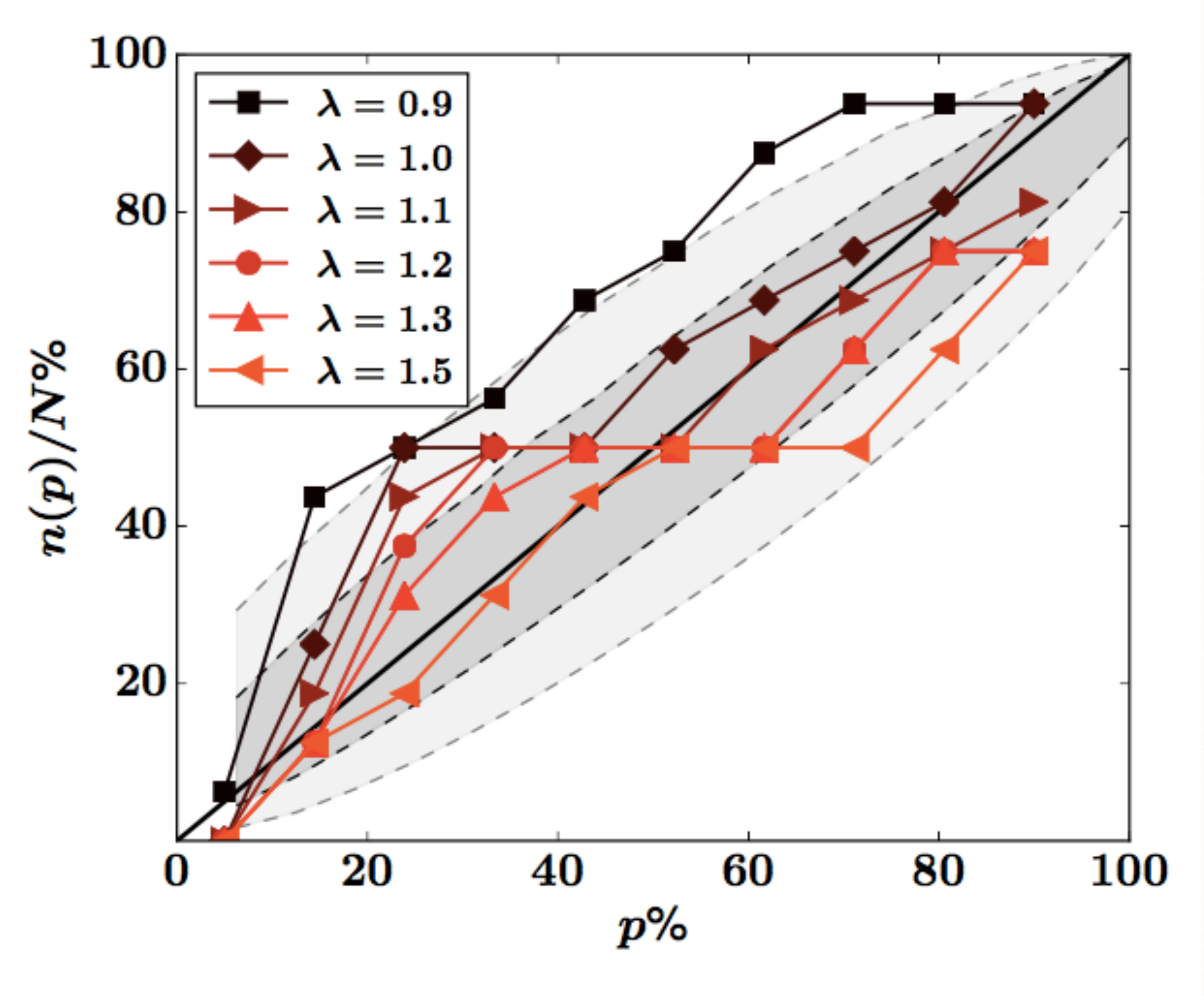


Source: "The Signal and the Noise" by Nate Silver | Author: Randy Olson (randalolson.com / @randal_olson)

The well-calibrated EFTist

Furnstahl, Klco, DP, Wesolowski, PRC, 2015

after: Bagnaschi, Cacciari, Guffanti, Jenniches, 2015



- Now we consider predictions at each order, with their error bars, as data and test them to see if the procedure is consistent

- Fix a given DOB interval, compute actual success ratio and compare

- Look at this over EKM predictions at four different orders and four different energies

- Interpret in terms of rescaling of Λ_b by a factor λ

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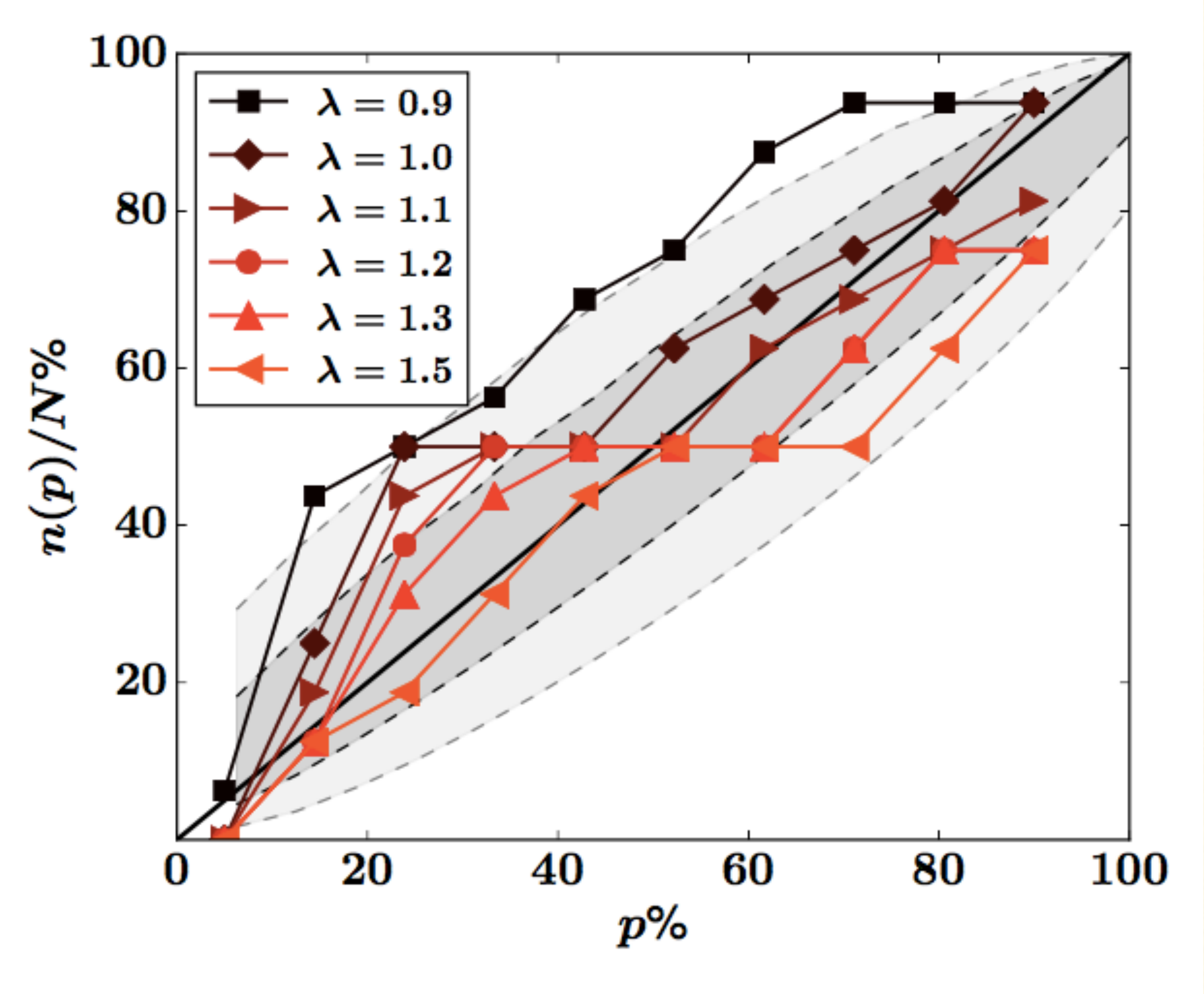
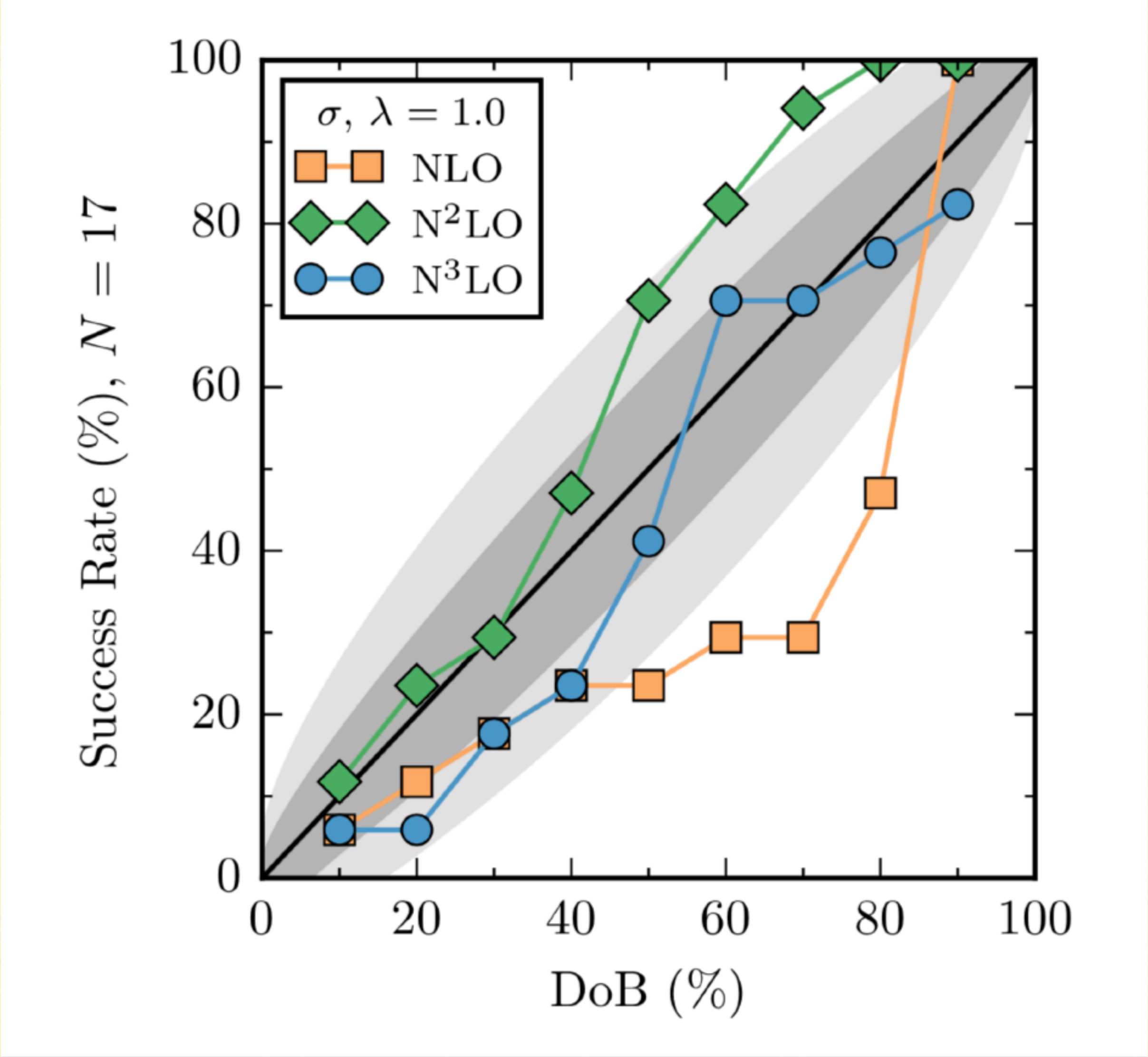


- Now we consider predictions at each order, with their error bars, as data and test them to see if the procedure is consistent
- Fix a given DOB interval, compute actual success ratio and compare
- Look at this over EKM predictions at four different orders and four different energies
- Interpret in terms of rescaling of Λ_b by a factor λ

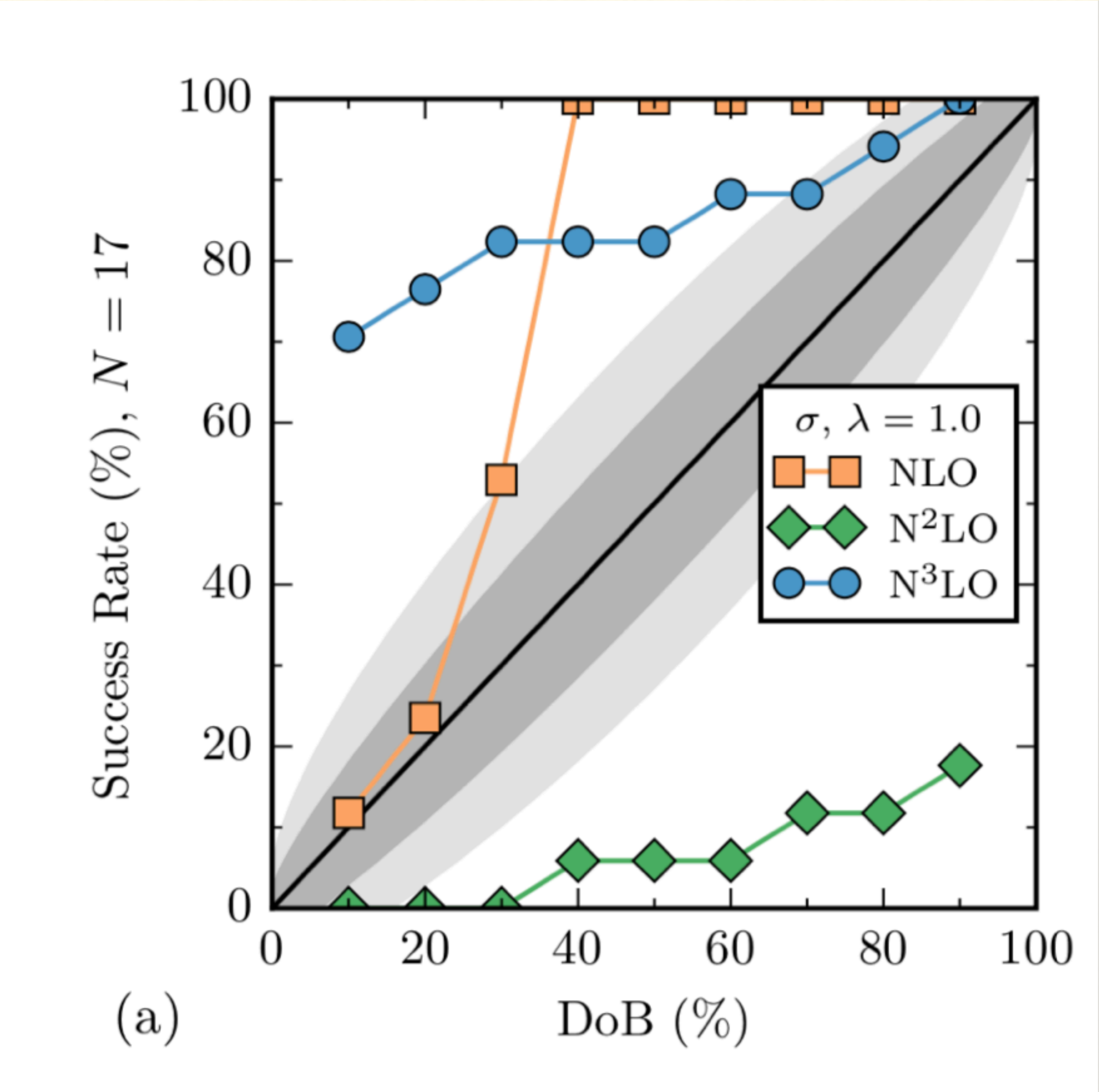
No evidence for significant rescaling of Λ_b

Physics from consistency plots

R=0.9 fm



R=1.2 fm



- Allows assessment of order-by-order convergence
- Can look at differential cross section and spin observables too

Physics from consistency plots

Melendez, Furnstahl, Wesolowski, PRC, 2017

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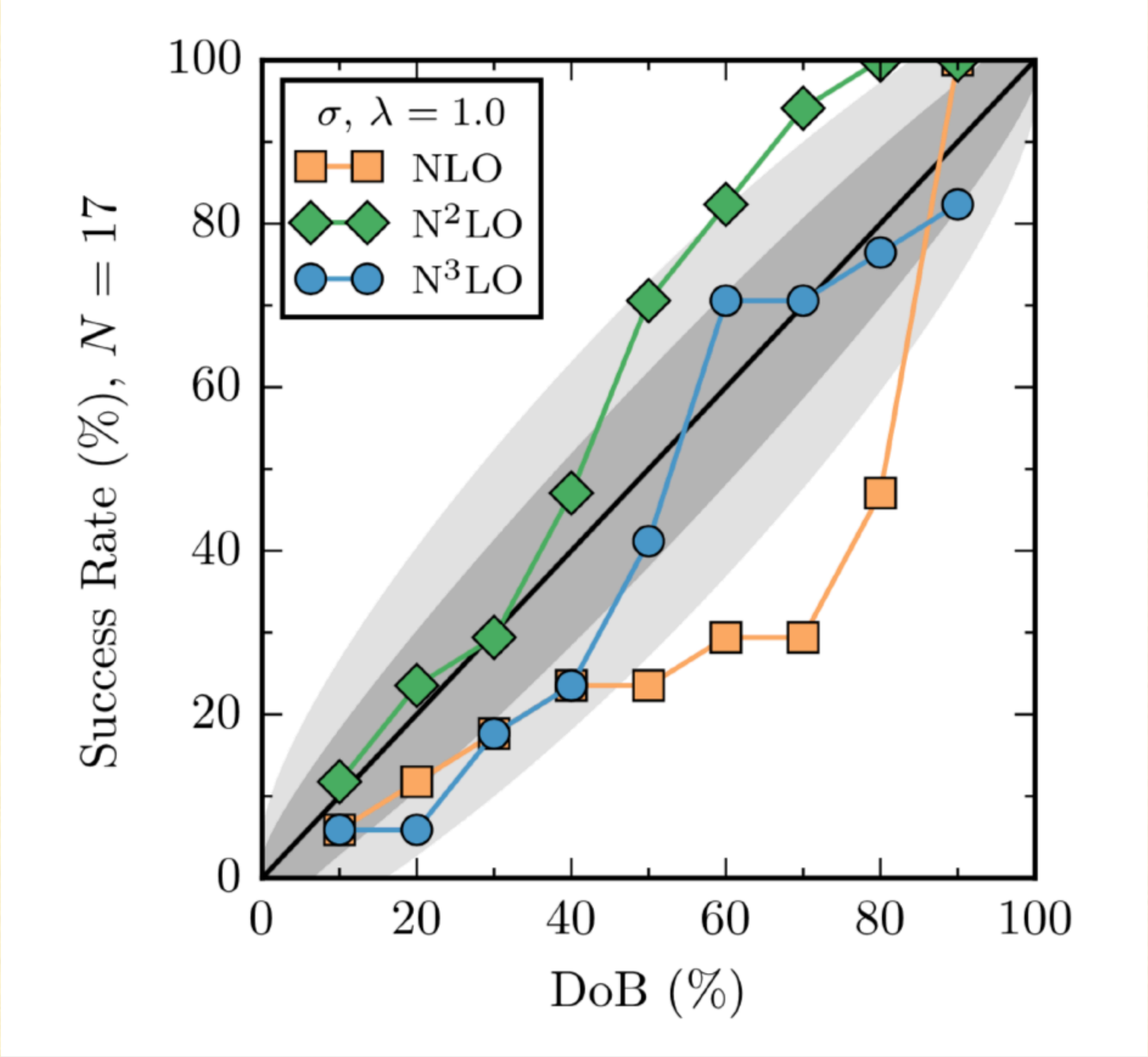

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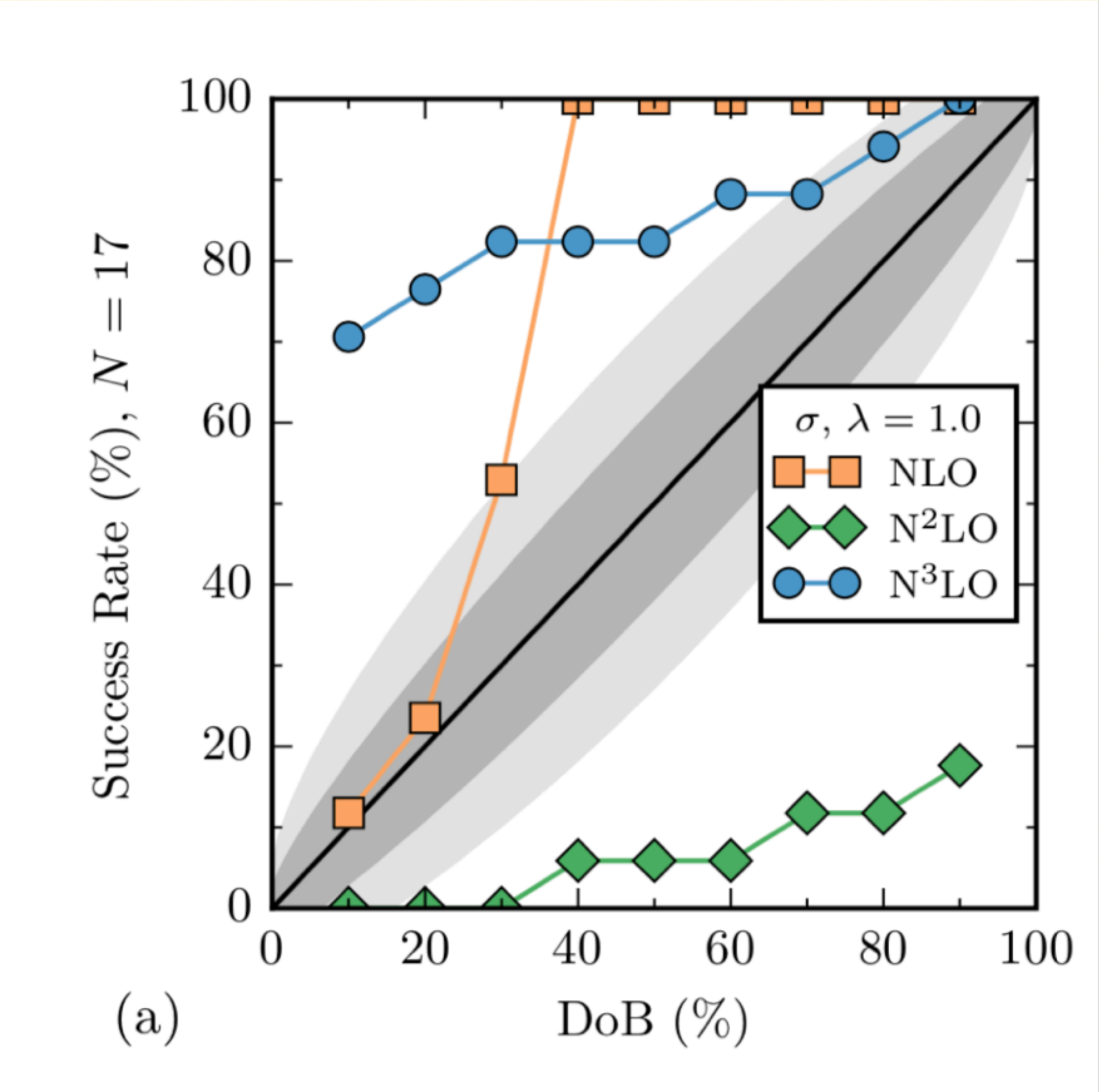

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Outline

- What we do and don't know about the strong nuclear force √
- EFT: organizing what we know, constraining what we don't √
- EFT truncation errors from a Bayesian analysis: NN scattering √
- EFT for halo nuclei: universal formula for $\gamma + {}^{A}Z \rightarrow {}^{A-1}Z + n$
- Uncertainty quantification for fusion: ^{7}Be(p,γ) at solar energies
- Conclusion

Ordinary vs. halo nuclei

Ordinary vs. halo nuclei

- In nuclei, each nucleon moves in the potential generated by the others
- The nuclear size grows as $A^{1/3}$; cross sections like $A^{2/3}$
- Nuclear binding energies are on the order of 8 MeV/nucleon

http://alternativephysics.org

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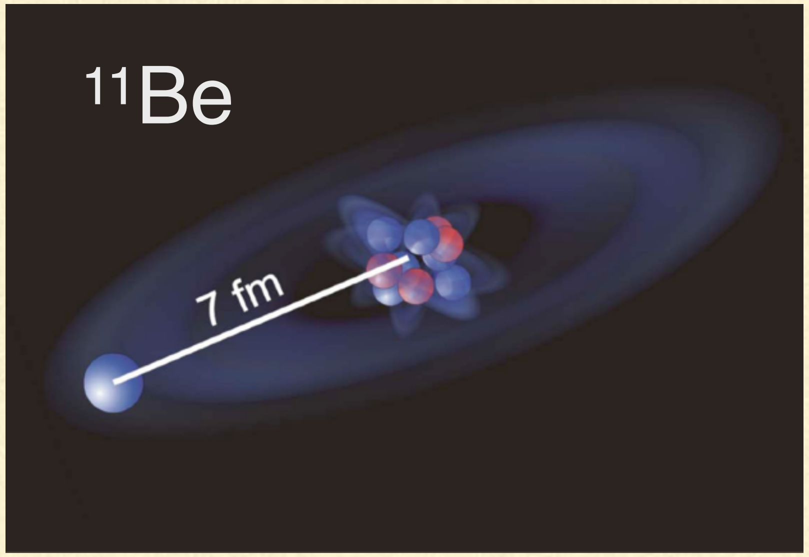

http://www.uni-mainz.de

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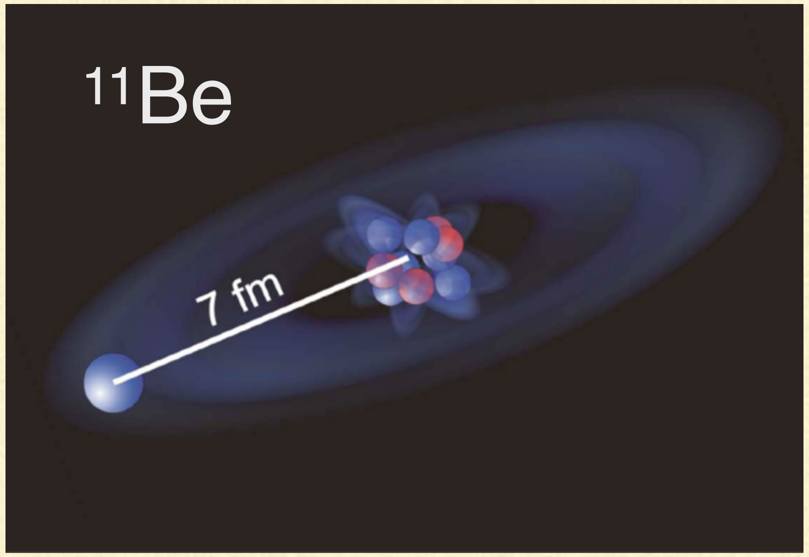


http://www.uni-mainz.de

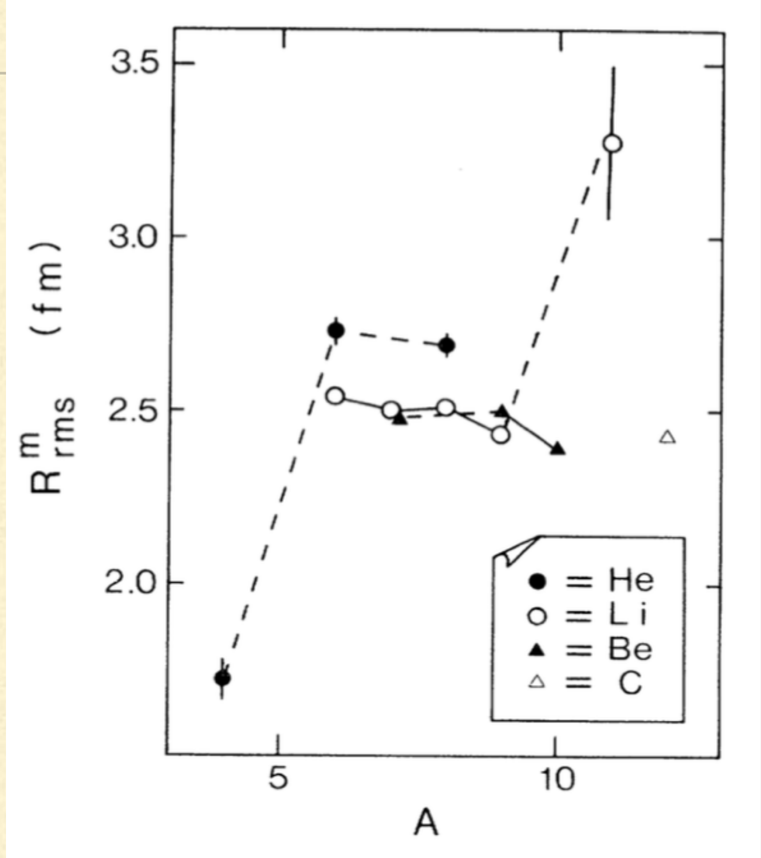
- Halo nuclei: the last few nucleons “orbit” far from the nuclear “core”
- Characterized by small nucleon binding energies, large radii, large interaction cross sections, large E1 transition strengths.

Halo nuclei: history & examples

Halo nuclei: history & examples

- ^{11}Li identified as “halo nucleus” in 1985

Tanihata et al., Phys. Rev. Lett. (1985)



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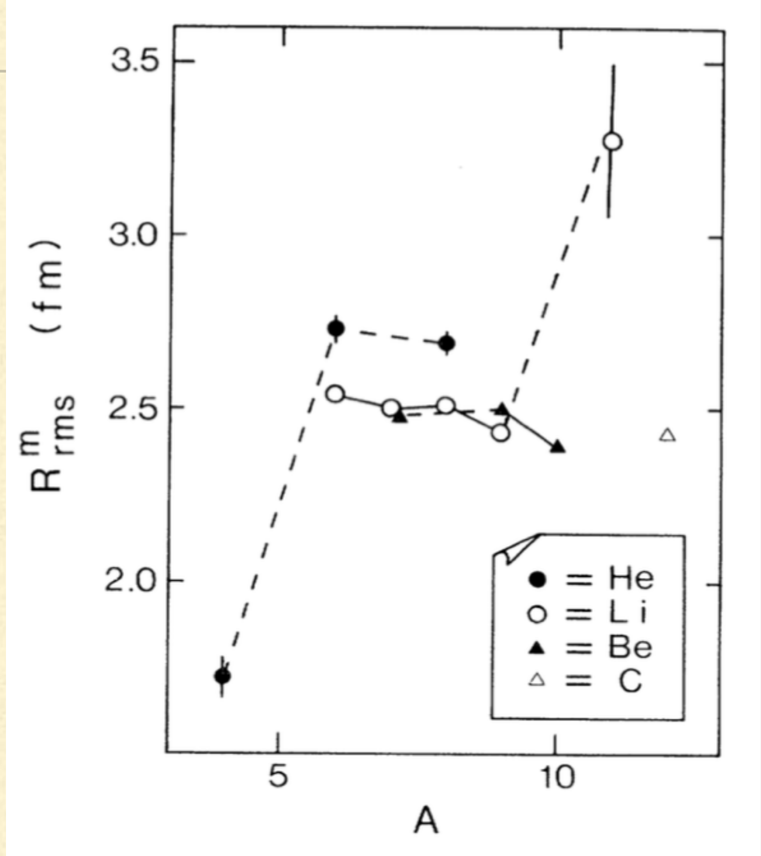

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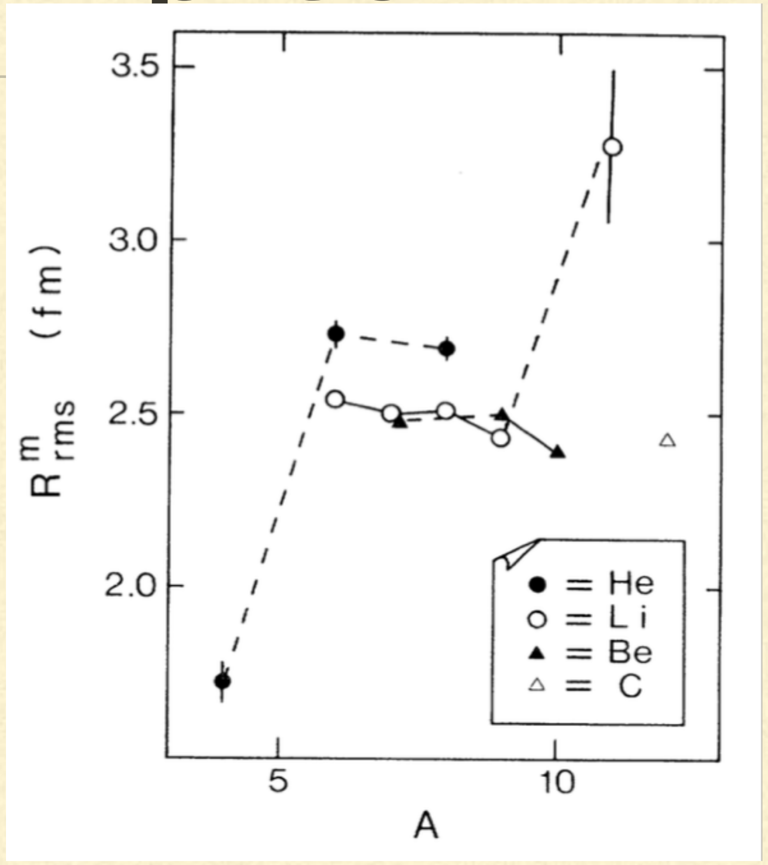

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- Understanding essential to modeling of neutron-rich nuclei

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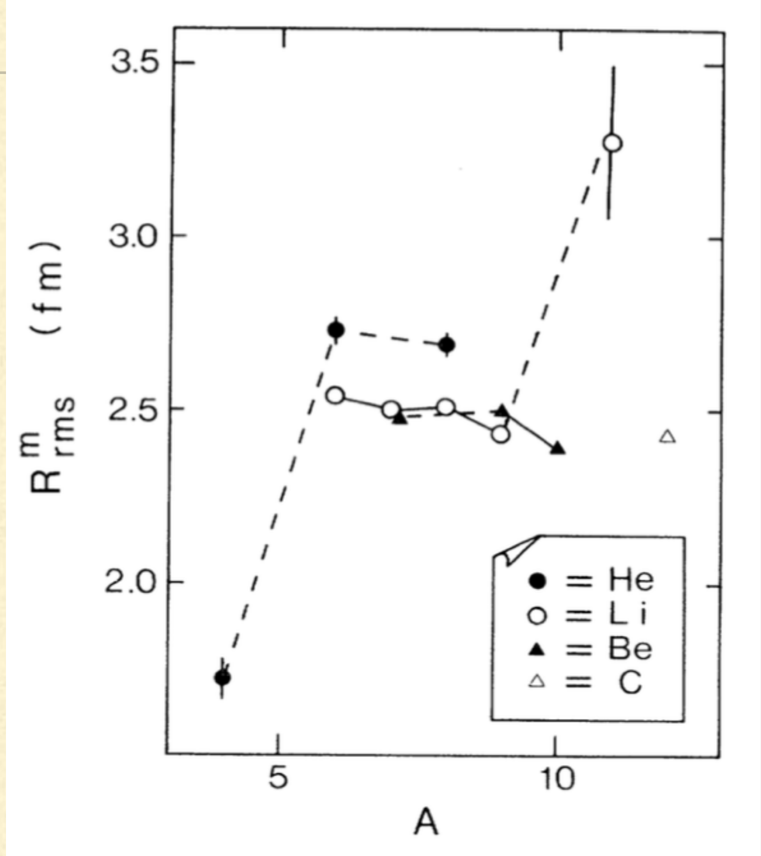

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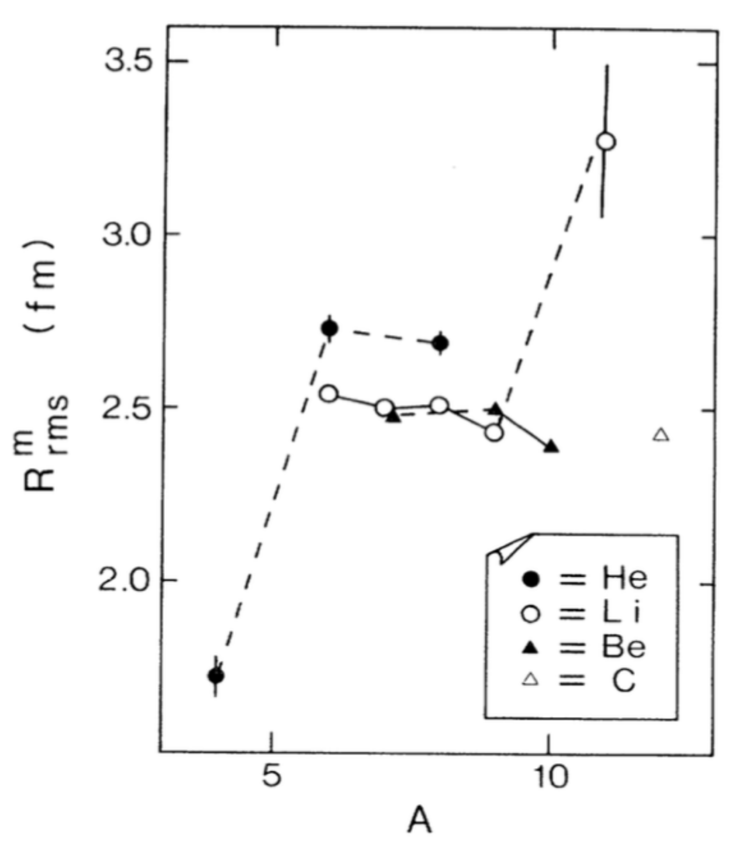

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- "Open quantum systems": physics beyond mean field

- Universality: common features of weakly-bound quantum systems

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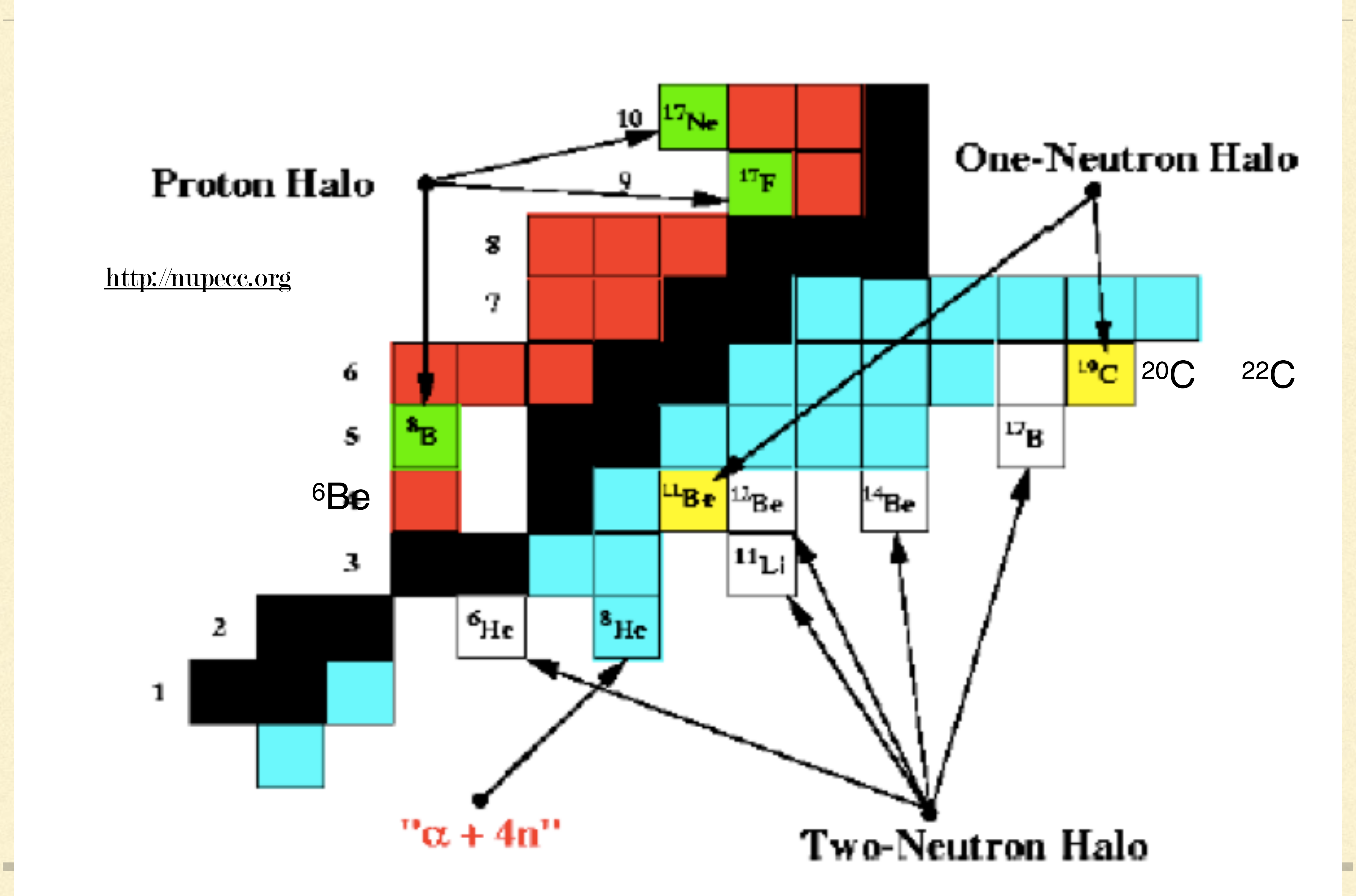

Halo EFT

Bertulani, Hammer, van Kolck, NPA (2003);
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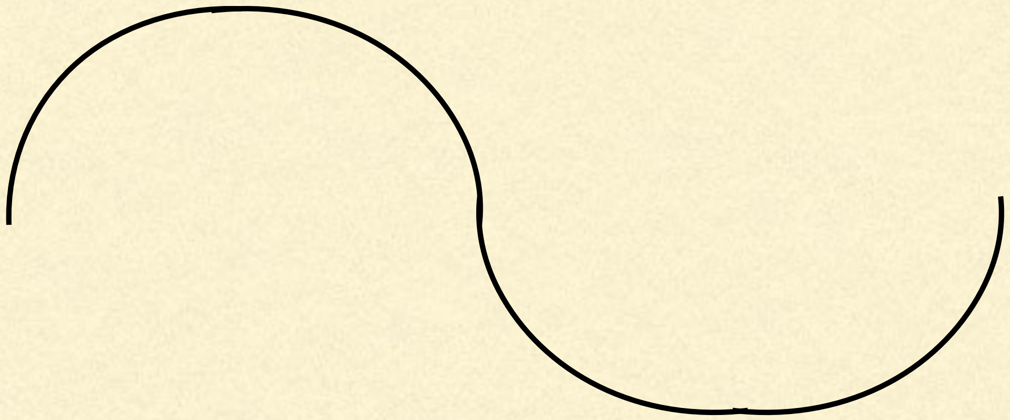
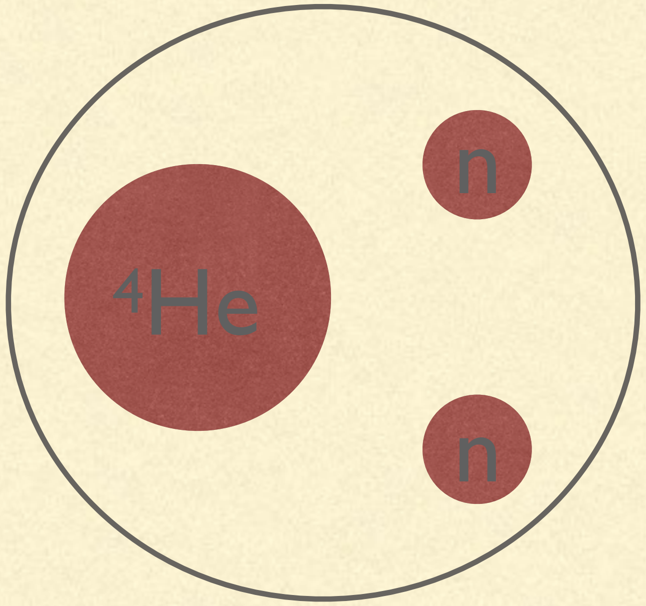
Review: Hammer, Ji, DP, J. Phys. G 44, 103002 (2017).

$\lambda \gg R_{core}; \ \lambda \lesssim R_{halo}$





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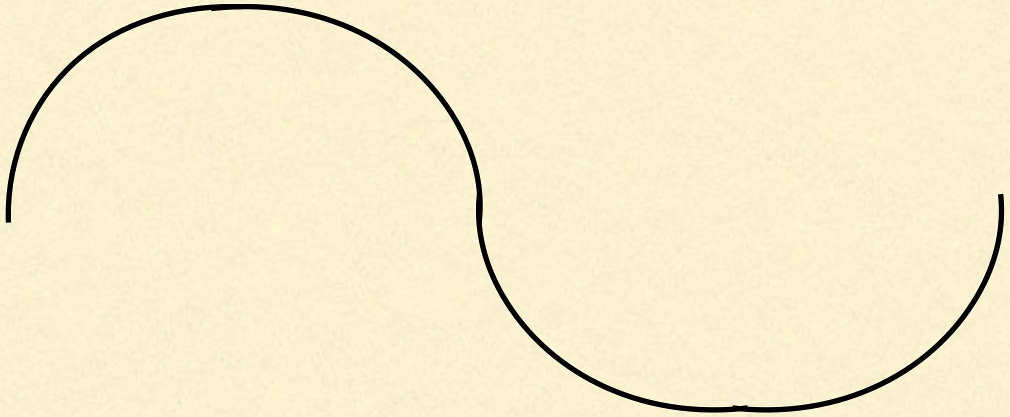
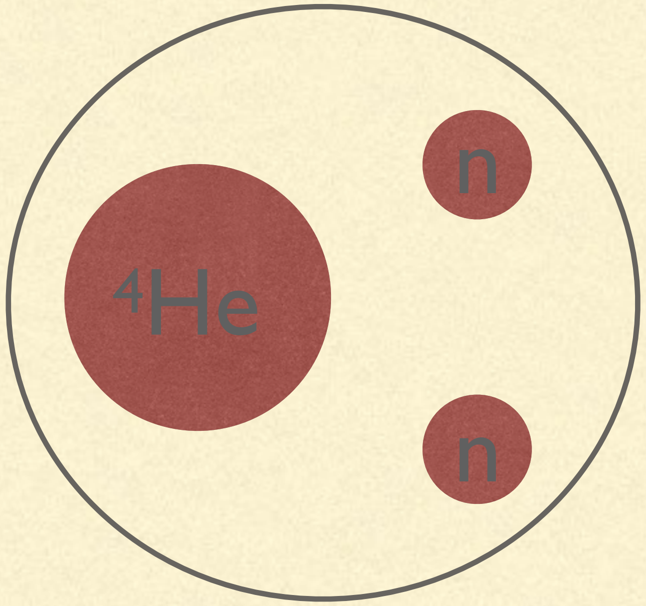




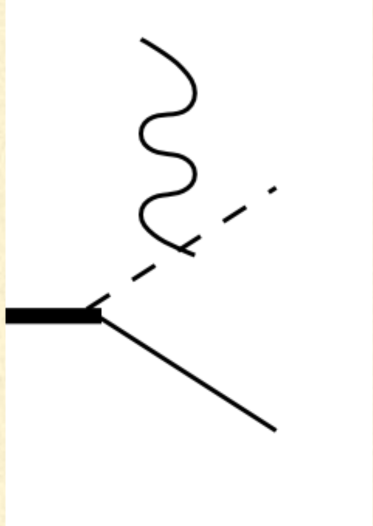
- Define $R_{halo}=\langle r^2 \rangle^{1/2}$. Seek EFT expansion in R_{core}/R_{halo}. Valid for $\lambda \lesssim R_{halo}$
- Typically $R \equiv R_{core} \sim 2$ fm. And since $\langle r^2 \rangle$ is related to the neutron separation energy we are looking for systems with neutron separation energies of order 1 MeV or less
- By this definition the deuteron is the lightest halo nucleus, and the pionless EFT for few-nucleon systems is a specific case of Halo EFT

Predicting dissociation

$$\mathcal{M} = \frac{eZg_0 2m_R}{\gamma_0^2 + \left(\mathbf{p} - \frac{\mathbf{k}}{A}\right)^2} \qquad \gamma_0 = \sqrt{2m_R S_{1n}}$$

$$p = \sqrt{2m_R E}$$

$$\mathrm{E1} \propto \int_0^\infty dr\, j_1(pr) r u_0(r); \quad u_0(r) = A_0 e^{-\gamma_0 r}$$



Chen, Savage (1999)

Predicting dissociation

- Leading order: no FSI⇒ γ_0 is only free parameter=0.16 fm^{-1} for ^{19}C

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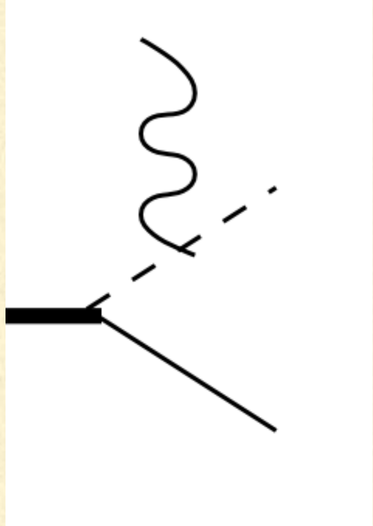

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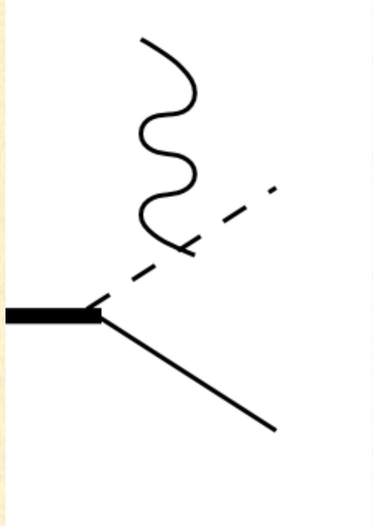

Chen, Savage (1999)

$$\frac{dB(E1)}{e^2 dE} = \frac{6m_R}{\pi^2}\frac{Z^2}{A^2} A_0^2 \frac{p^3}{(\gamma_0^2 + p^2)^2}$$

Universal E1 strength formula for S-wave halos

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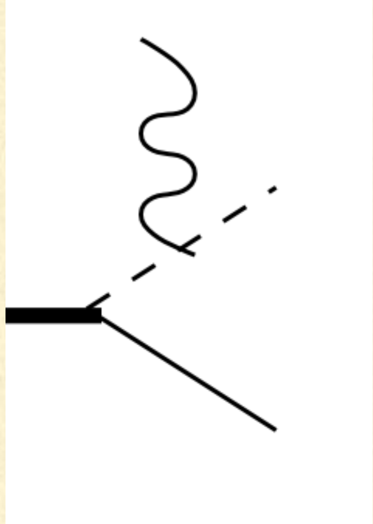

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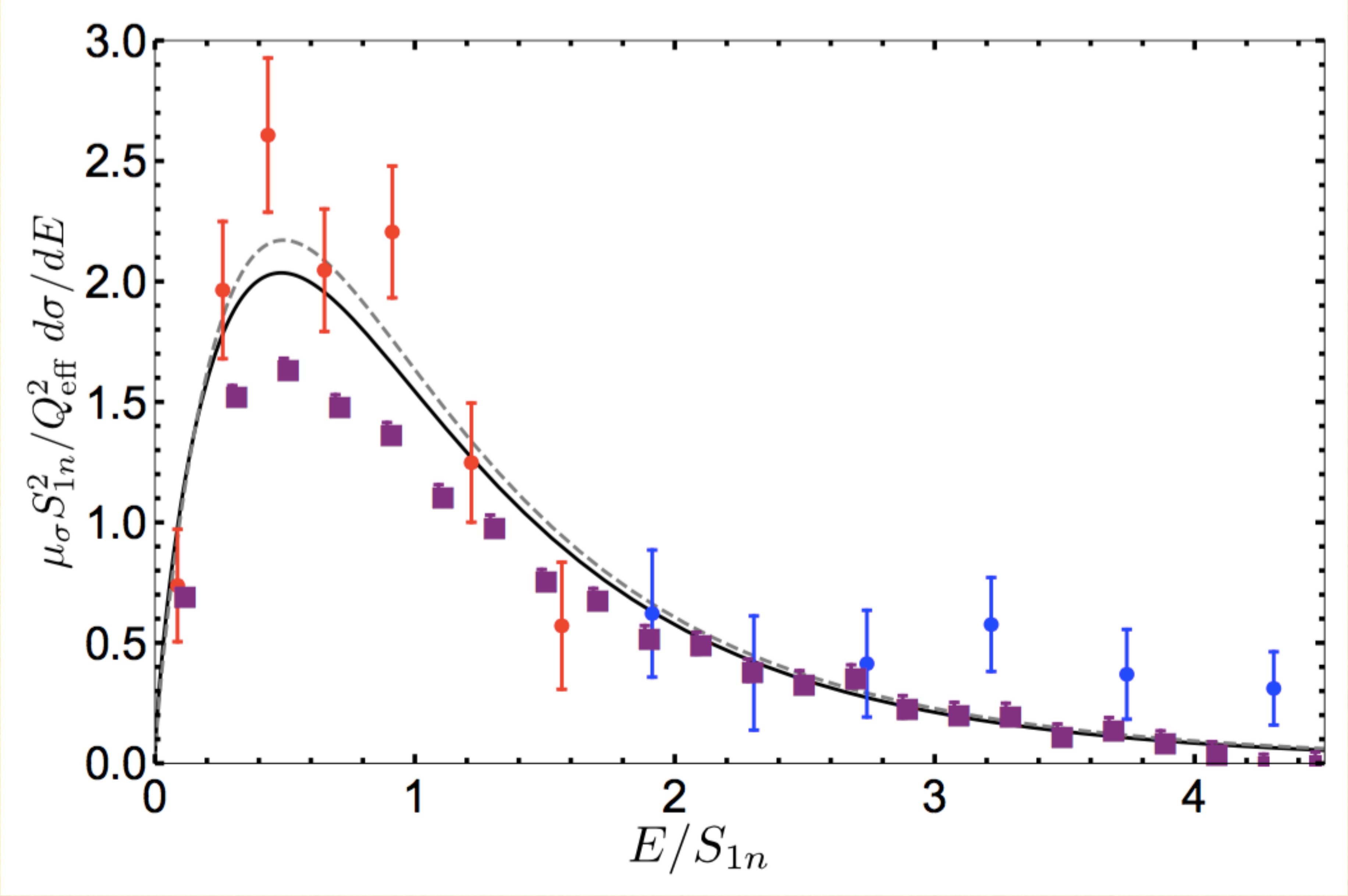
Universal E1 strength formula for S-wave halos

- Final-state interactions suppressed by $(R_{\mathrm{core}}/R_{\mathrm{halo}})^3$
- Short-distance piece of E1 m.e.: $L_{E1} \sigma^\dagger \mathbf{E} \cdot (n \overset{\leftrightarrow}{\nabla} c) + \mathrm{h.c.} \sim \left(\frac{R_{\mathrm{core}}}{R_{\mathrm{halo}}}\right)^4$

Results

Results



Data: Nakamura et al., 1999, 2003;
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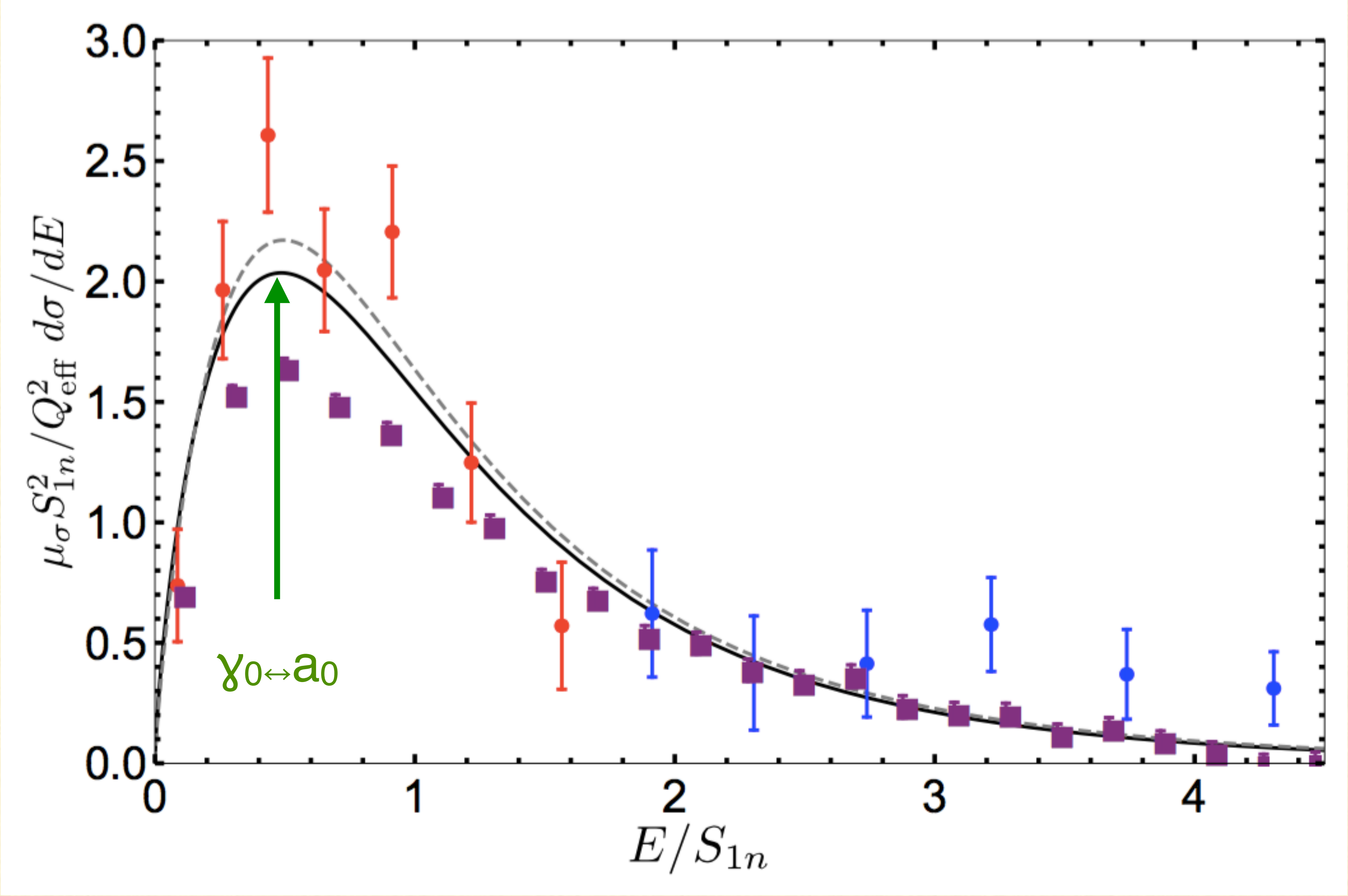

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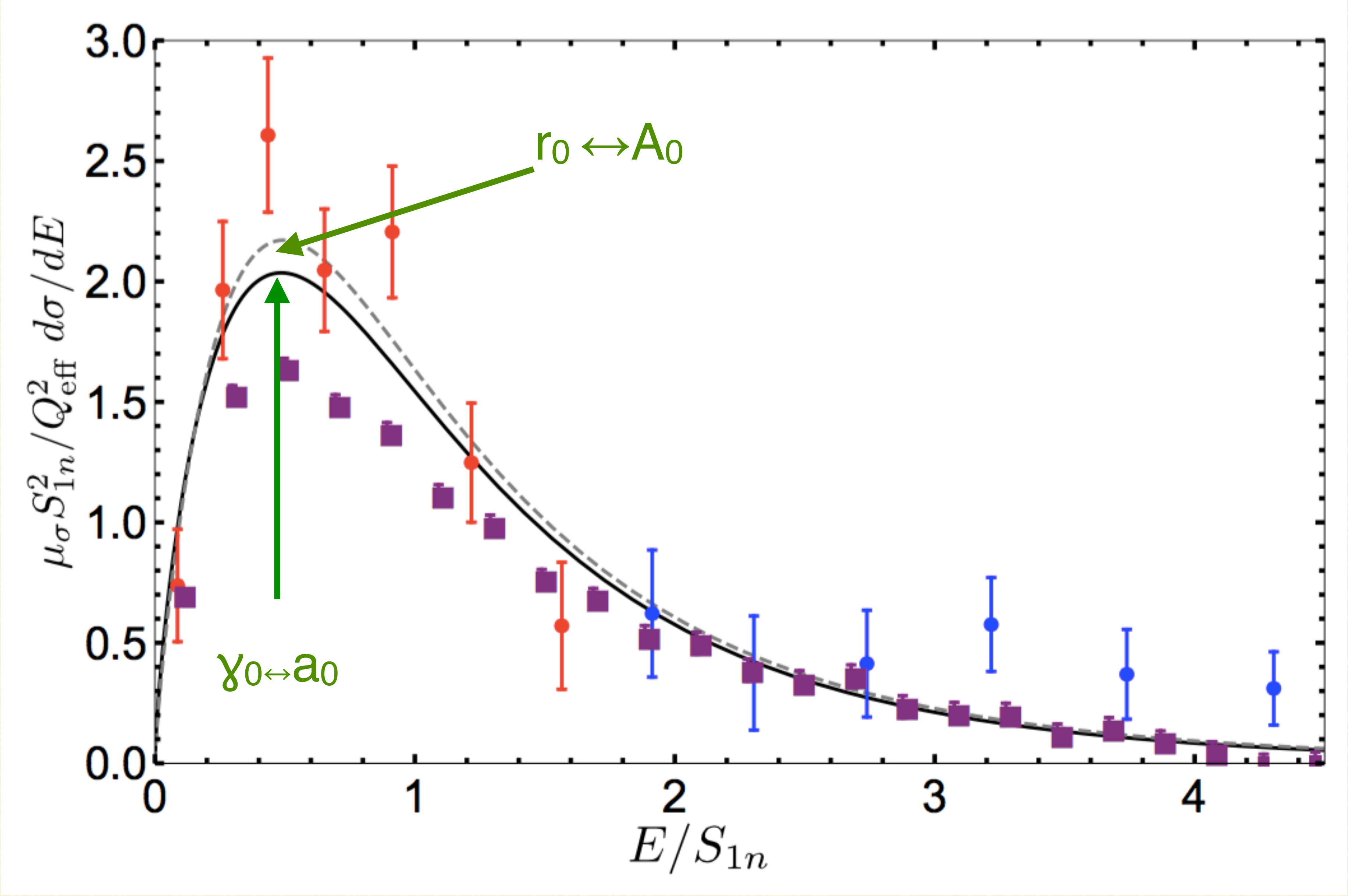

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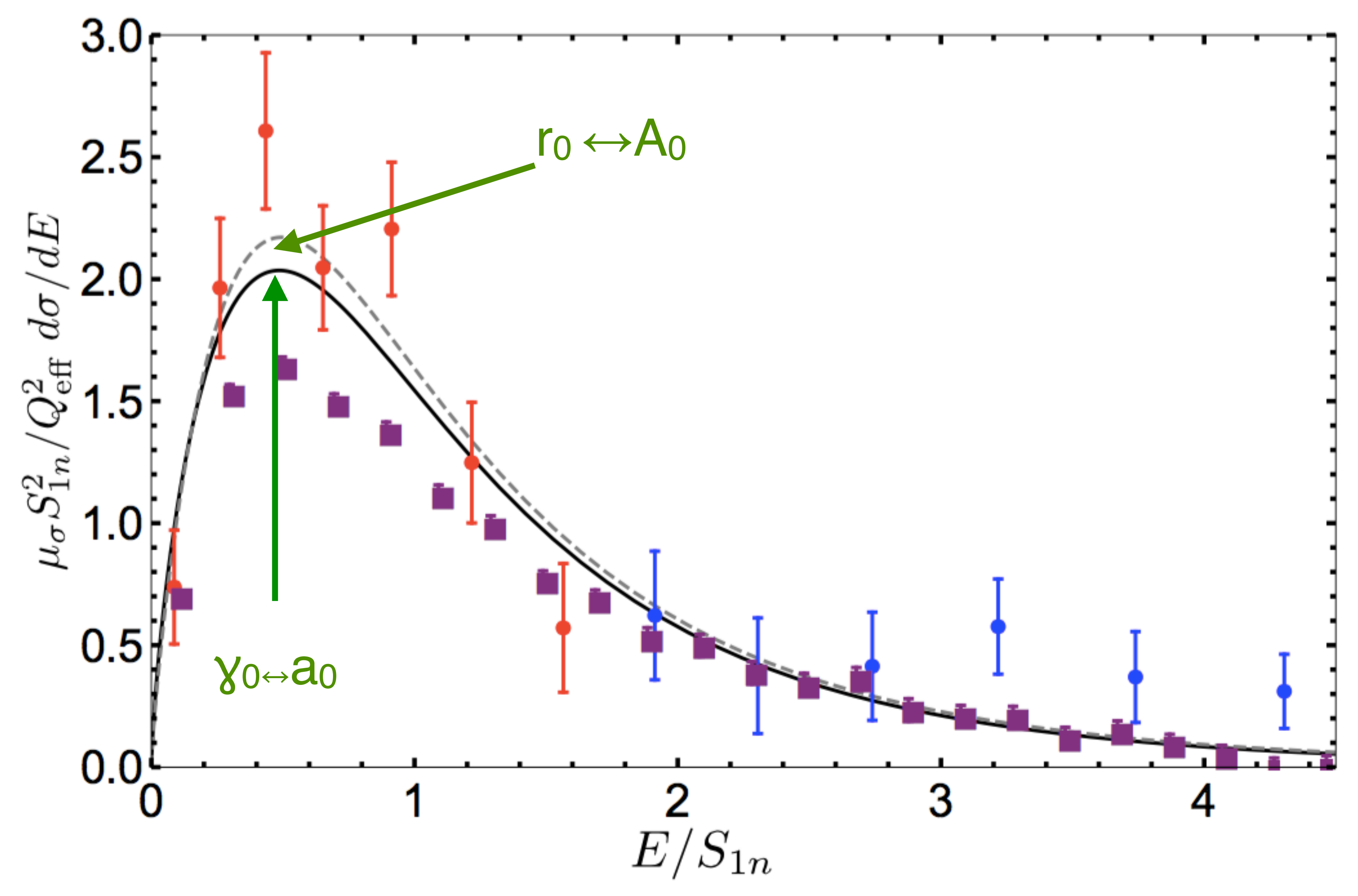

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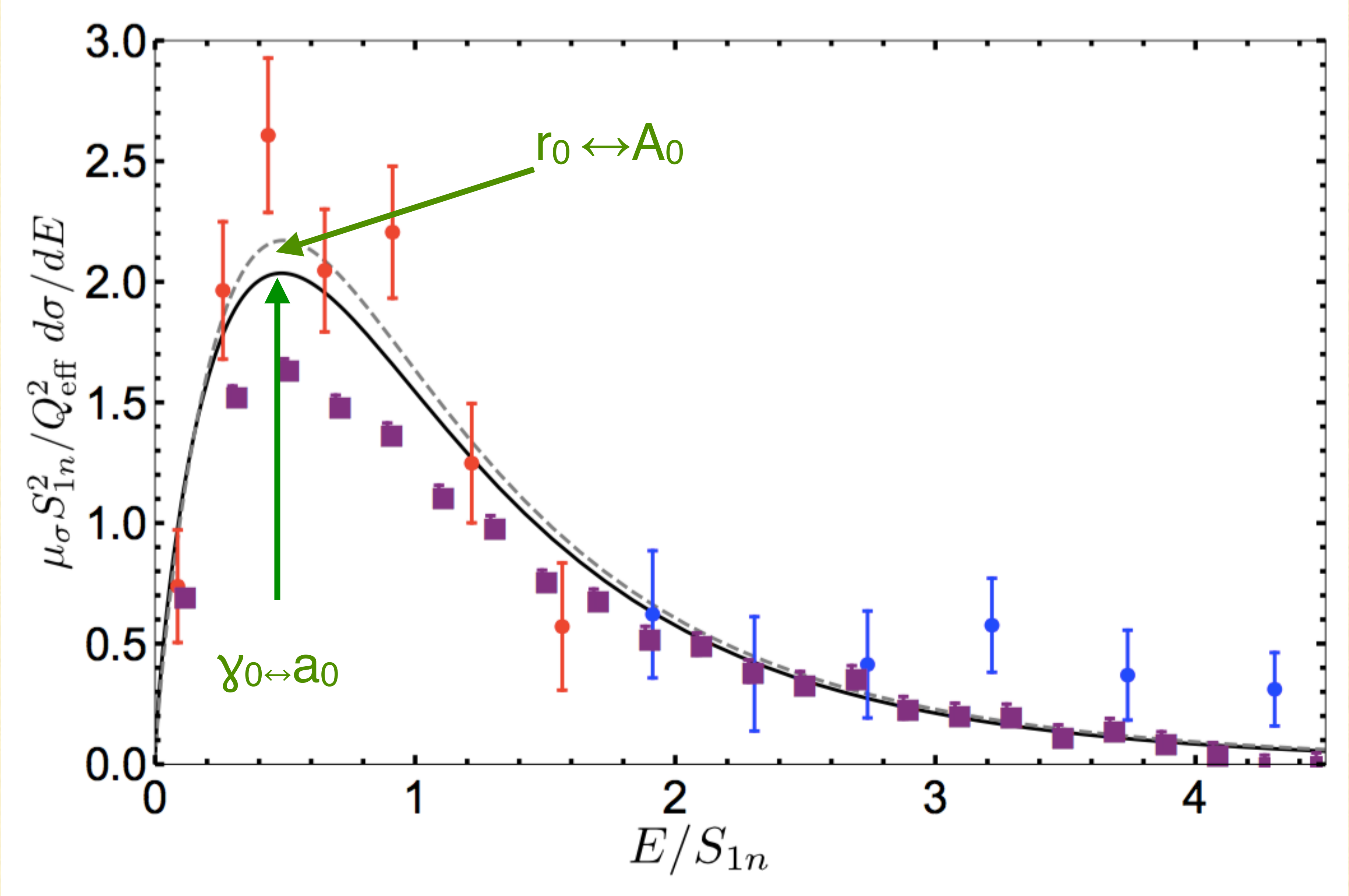
Determine S-wave ^{18}C-n scattering parameters⇔^{19}C ANC from dissociation data.

Results



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Analysis: Acharya, Phillips, 2013;
Hammer, Ji, Phillips, 2017

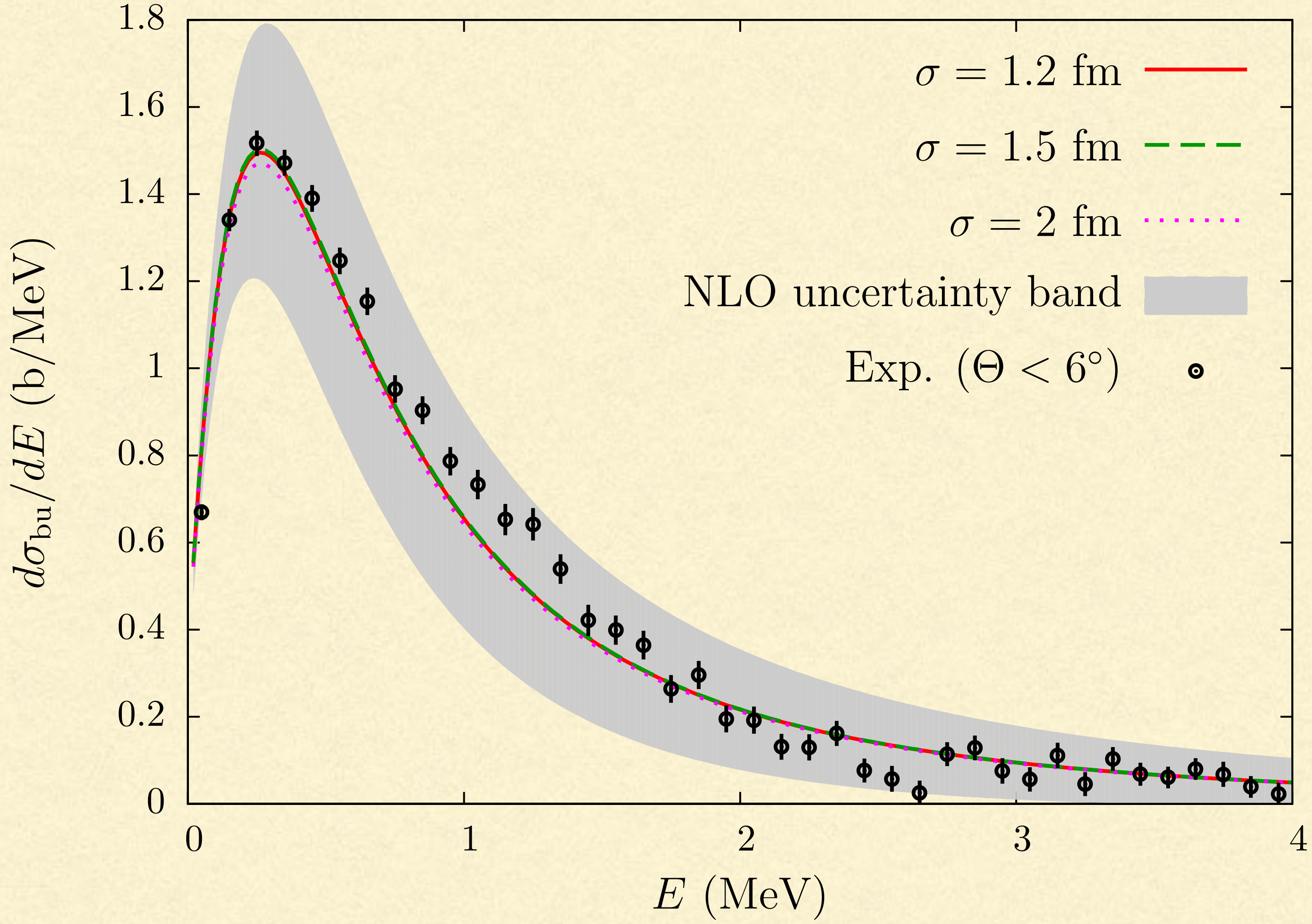
- For ^{19}C:

$$\begin{aligned} a &= (7.75 \pm 0.35(\text{stat.}) \pm 0.3(\text{EFT}))\ \text{fm}; \\ r_0 &= (2.6^{+0.6}_{-0.9}(\text{stat.}) \pm 0.1(\text{EFT}))\ \text{fm}. \end{aligned}$$

Ab initio→Halo EFT→Reaction theory

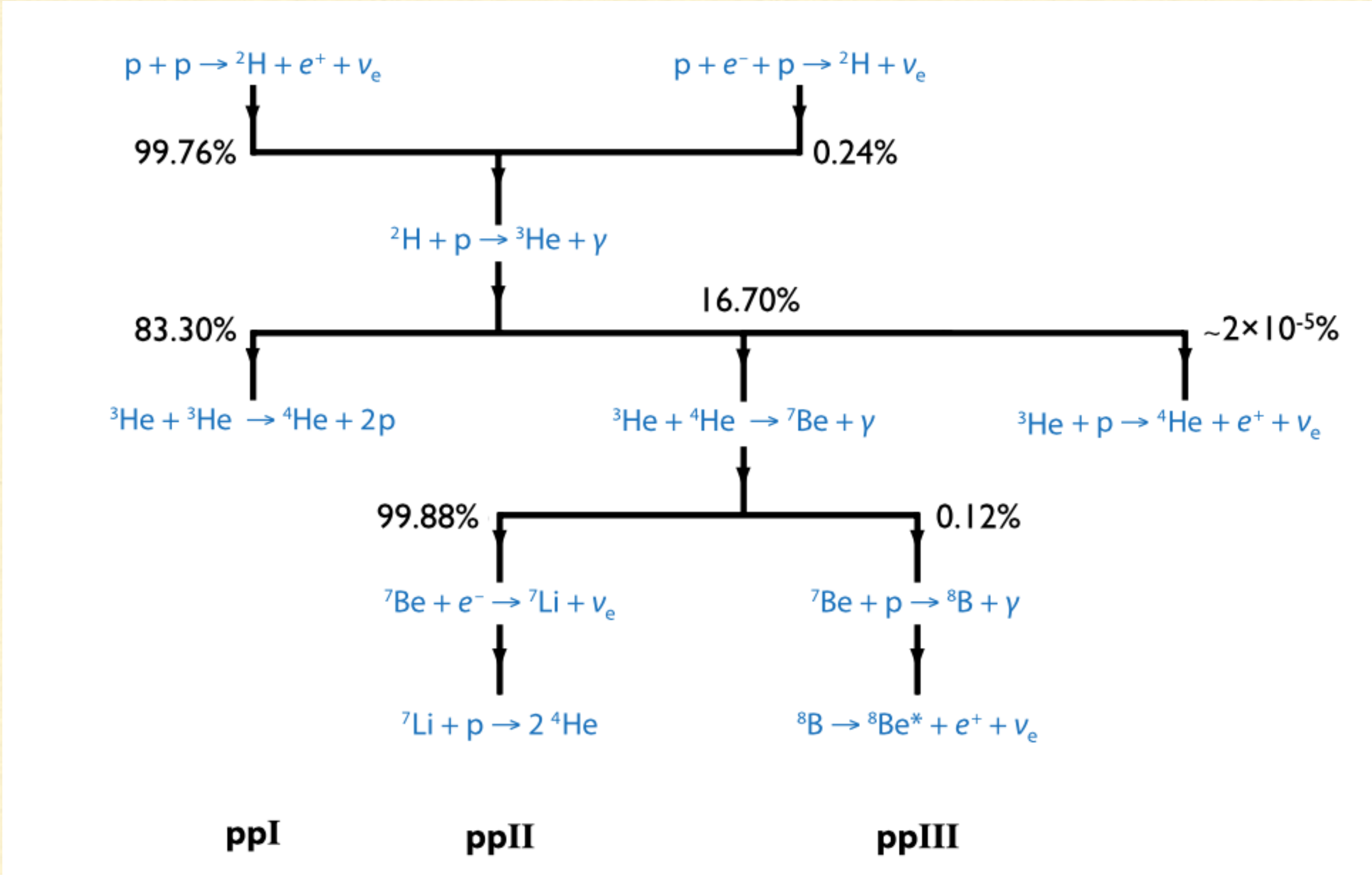
- ^{11}Be is a halo nucleus: last neutron only bound to ^{10}Be by 503 keV. Has a p-wave halo state with S_{1n}=184 keV.
- Model Coulomb dissociation of ^{11}Be via sophisticated "Dynamical Eikonal Approximation": includes nuclear and Coulomb ^{208}Pb-^{10}Be-n potentials
- Use Halo EFT to identify important ^{10}Be-n inputs for reaction-theory calculation: s- and p-wave phase shifts
- Take those from *ab initio* calculation of Calci et al. based on modern nuclear forces and NCSMC (PRL **117**, 242501)

Capel, DP, Hammer, Phys. Rev. C 98, 034610 (2018)
Data: Fukuda et al., Phys. Rev. C 70, 054606 (2004).



No dependence on interior of ^{10}Be-n potential used

Why is ^{7}Be(p,γ) important?



$p + p \rightarrow {}^2H + e^+ + \nu_e$

$p + e^- + p \rightarrow {}^2H + \nu_e$

99.76%

0.24%

${}^2H + p \rightarrow {}^3He + \gamma$

83.30%

16.70%

~2×10^{-5}%

${}^3He + {}^3He \rightarrow {}^4He + 2p$

${}^3He + {}^4He \rightarrow {}^7Be + \gamma$

${}^3He + p \rightarrow {}^4He + e^+ + \nu_e$

99.88%

0.12%

${}^7Be + e^- \rightarrow {}^7Li + \nu_e$

${}^7Be + p \rightarrow {}^8B + \gamma$

${}^7Li + p \rightarrow 2\,{}^4He$

${}^8B \rightarrow {}^8Be^* + e^+ + \nu_e$

ppI

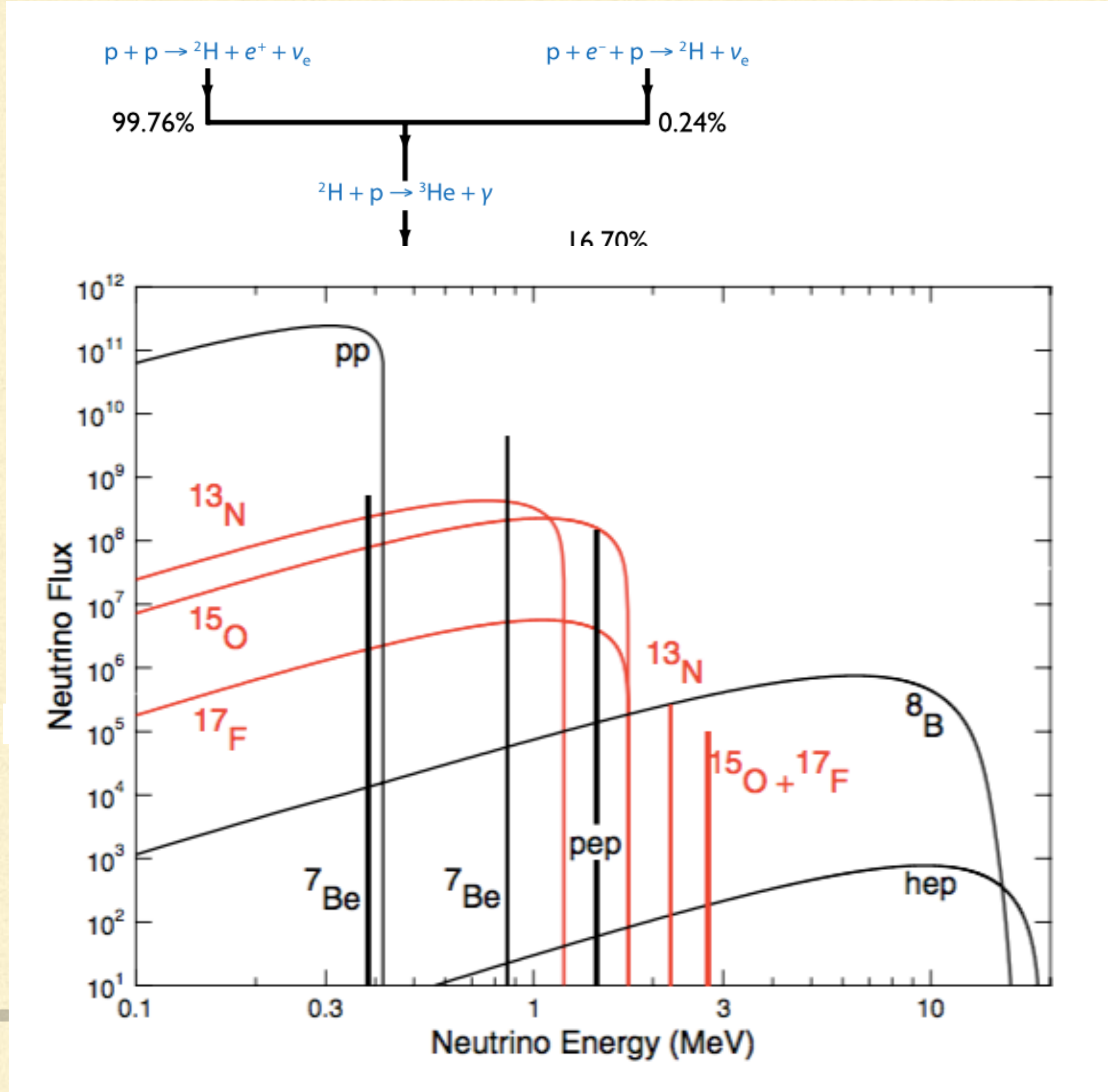
ppII

ppIII

Why is ^{7}Be(p,γ) important?

- Part of pp chain (ppIII)
- Key for predictions flux of solar neutrinos, especially high-energy (^{8}B) neutrinos
- Accurate knowledge of ^{7}Be(p,γ) needed for inferences from solar-neutrino flux regarding chemical composition of Sun→solar-system formation history
- S(0)=20.8 ± 0.7 ± 1.4 eV b

"SFII": Adelberger et al. (2010)



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Thermonuclear reaction rate $\propto \langle v\sigma \rangle \propto \int_0^\infty dE\, \exp\left(-\frac{E}{k_B T}\right) E\, \sigma(E)$

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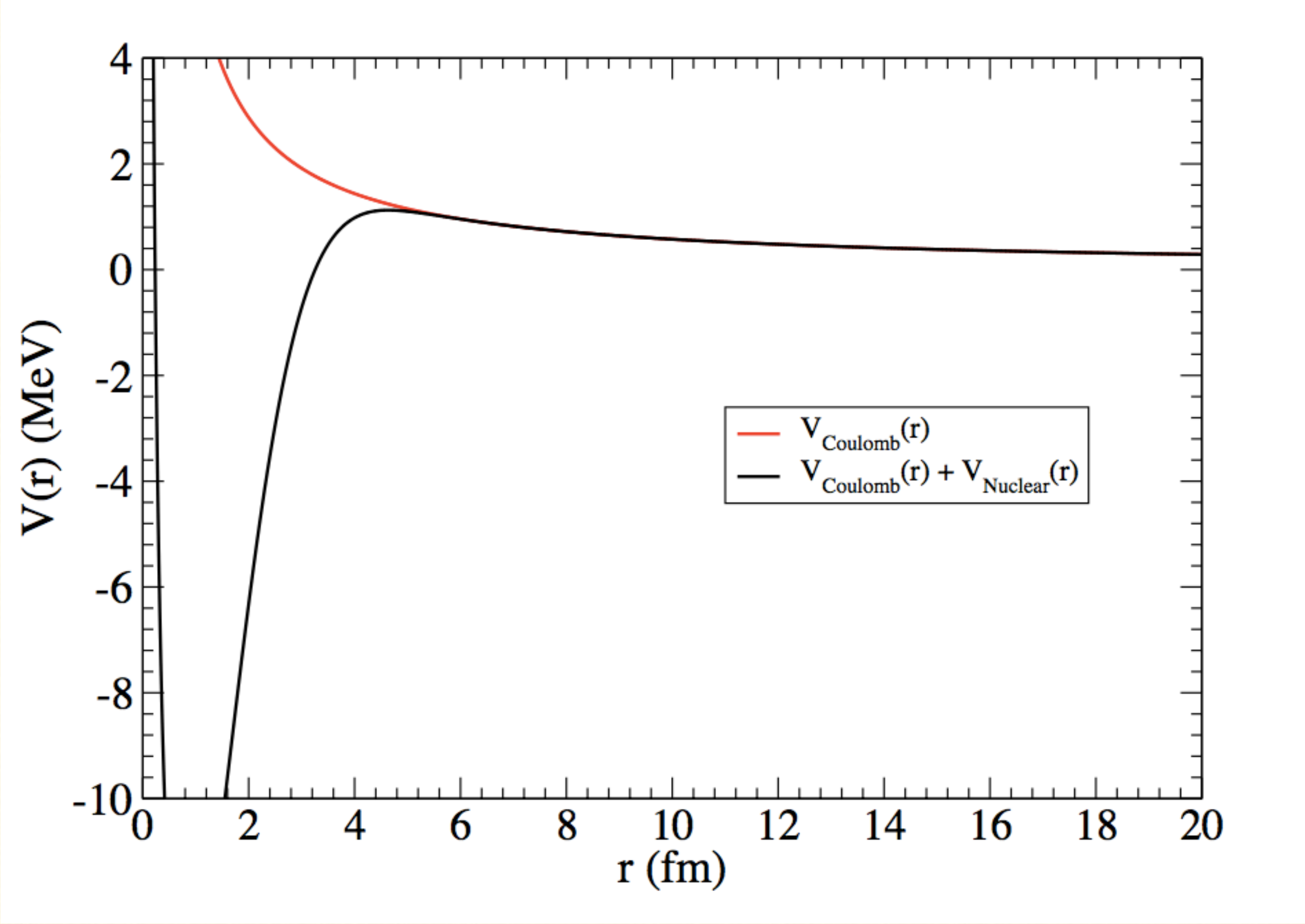

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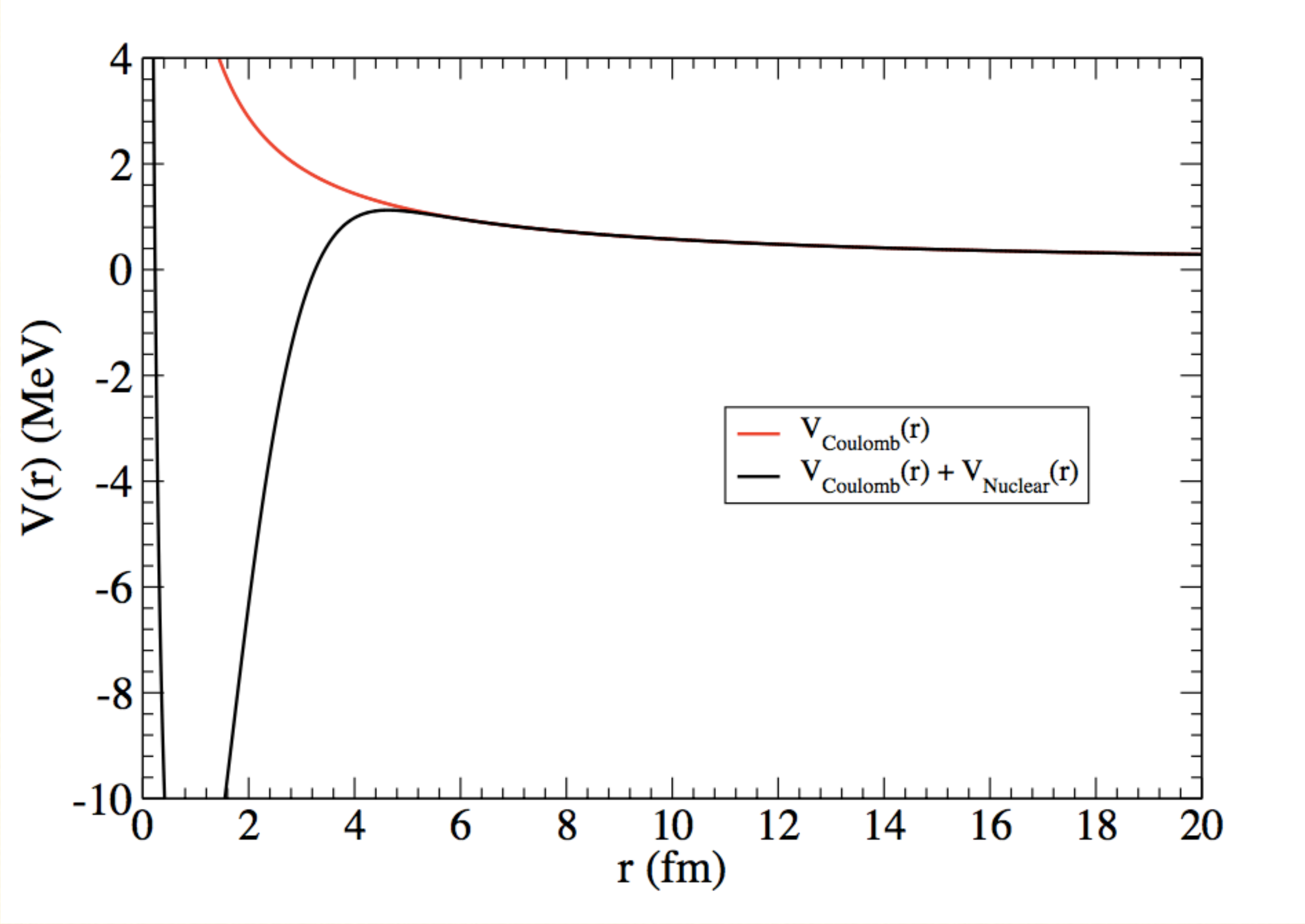

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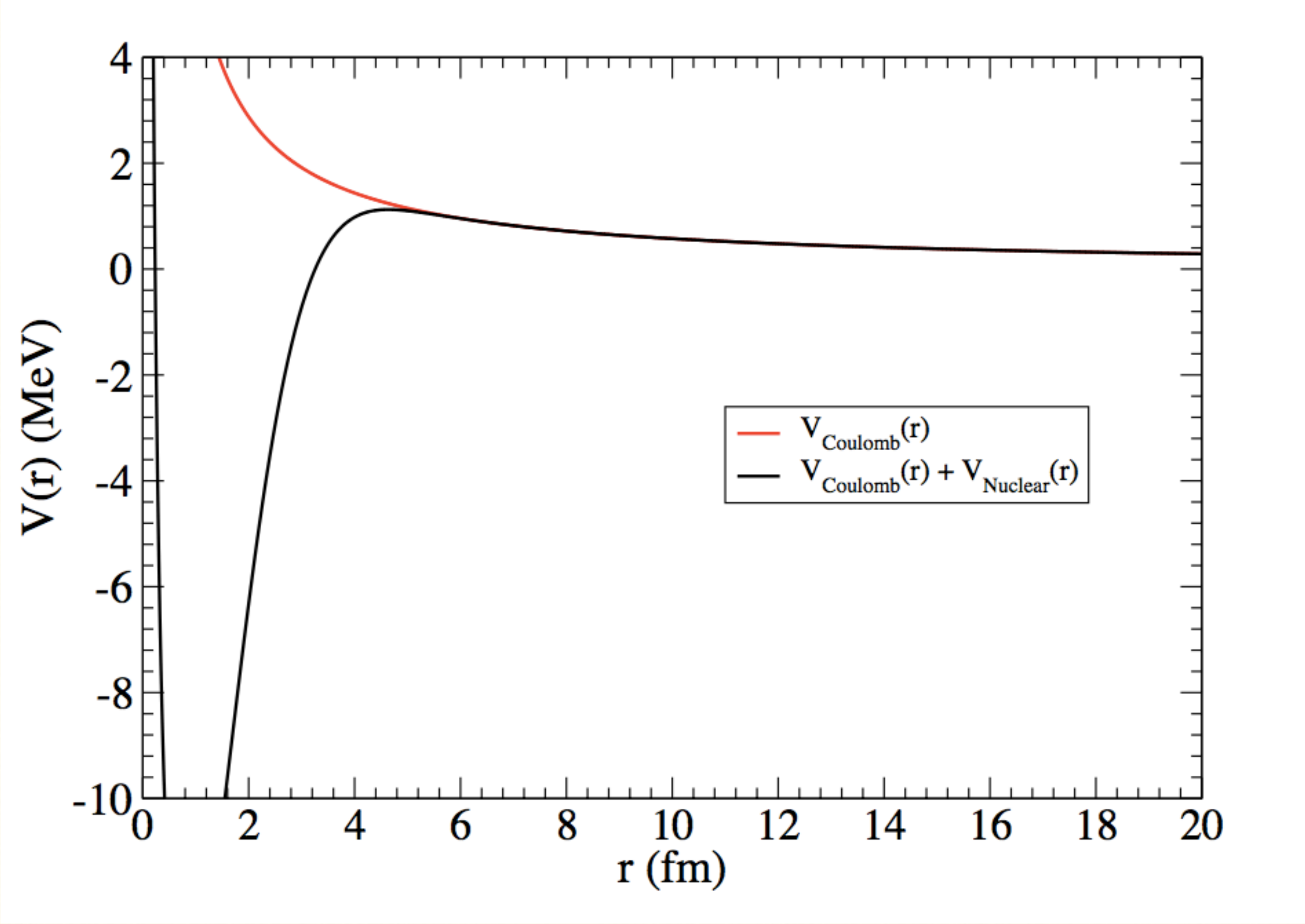

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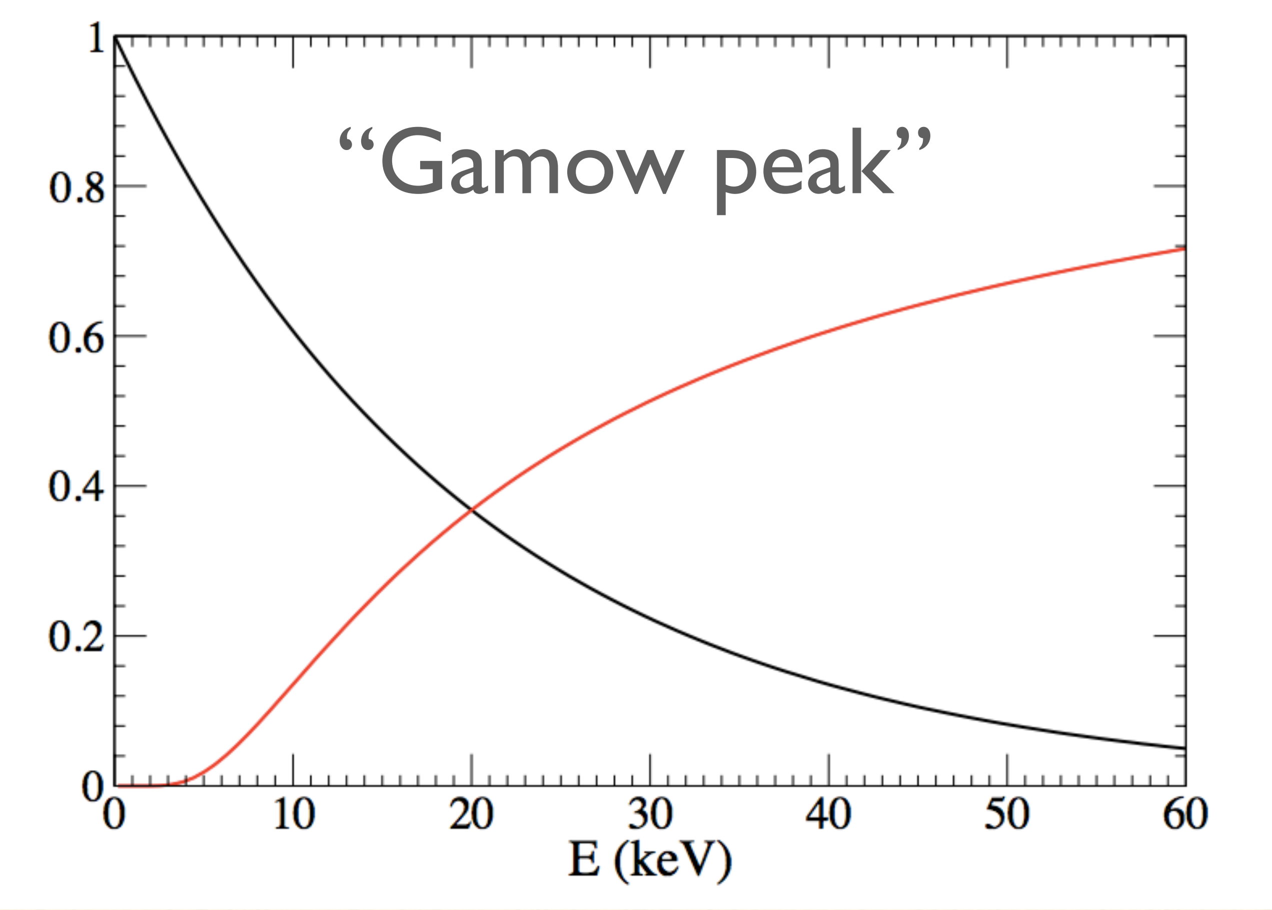

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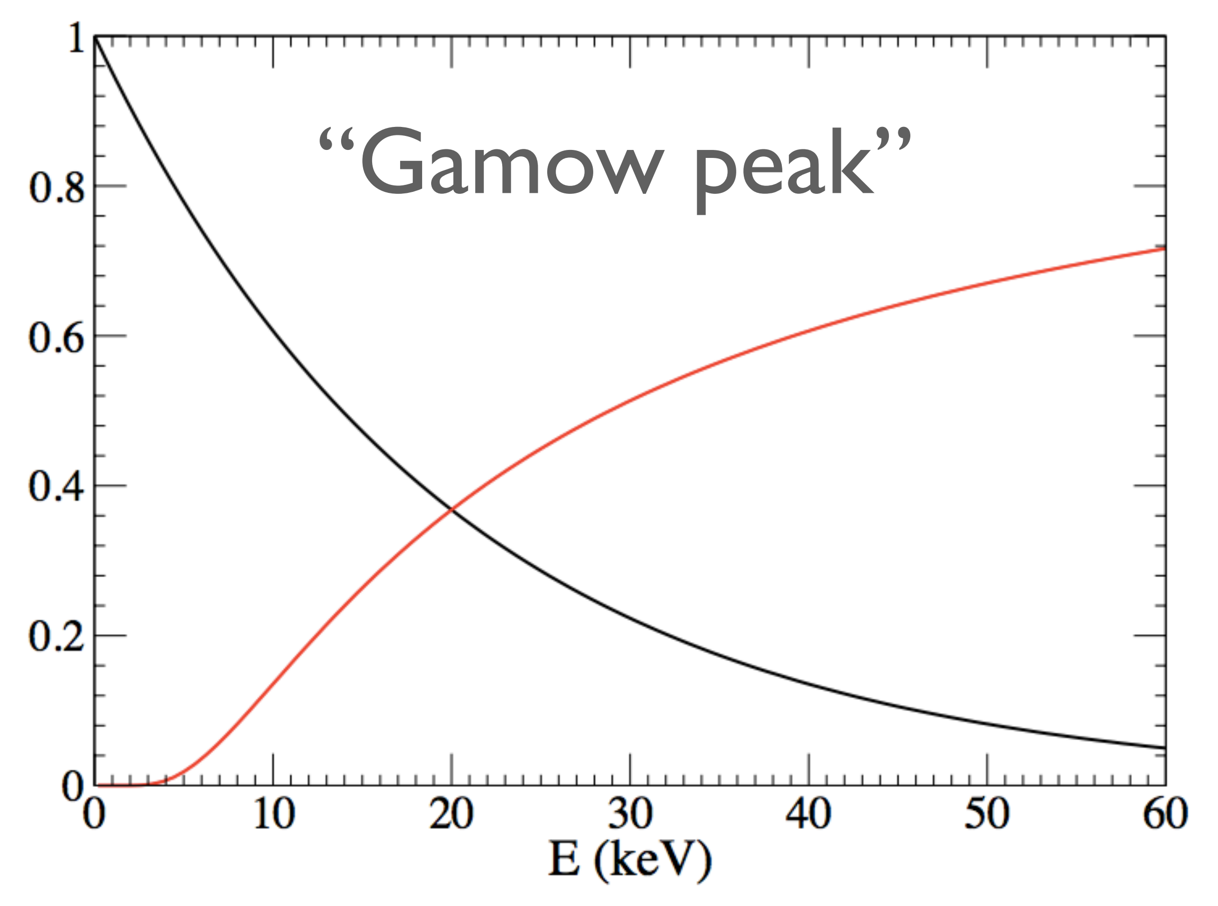
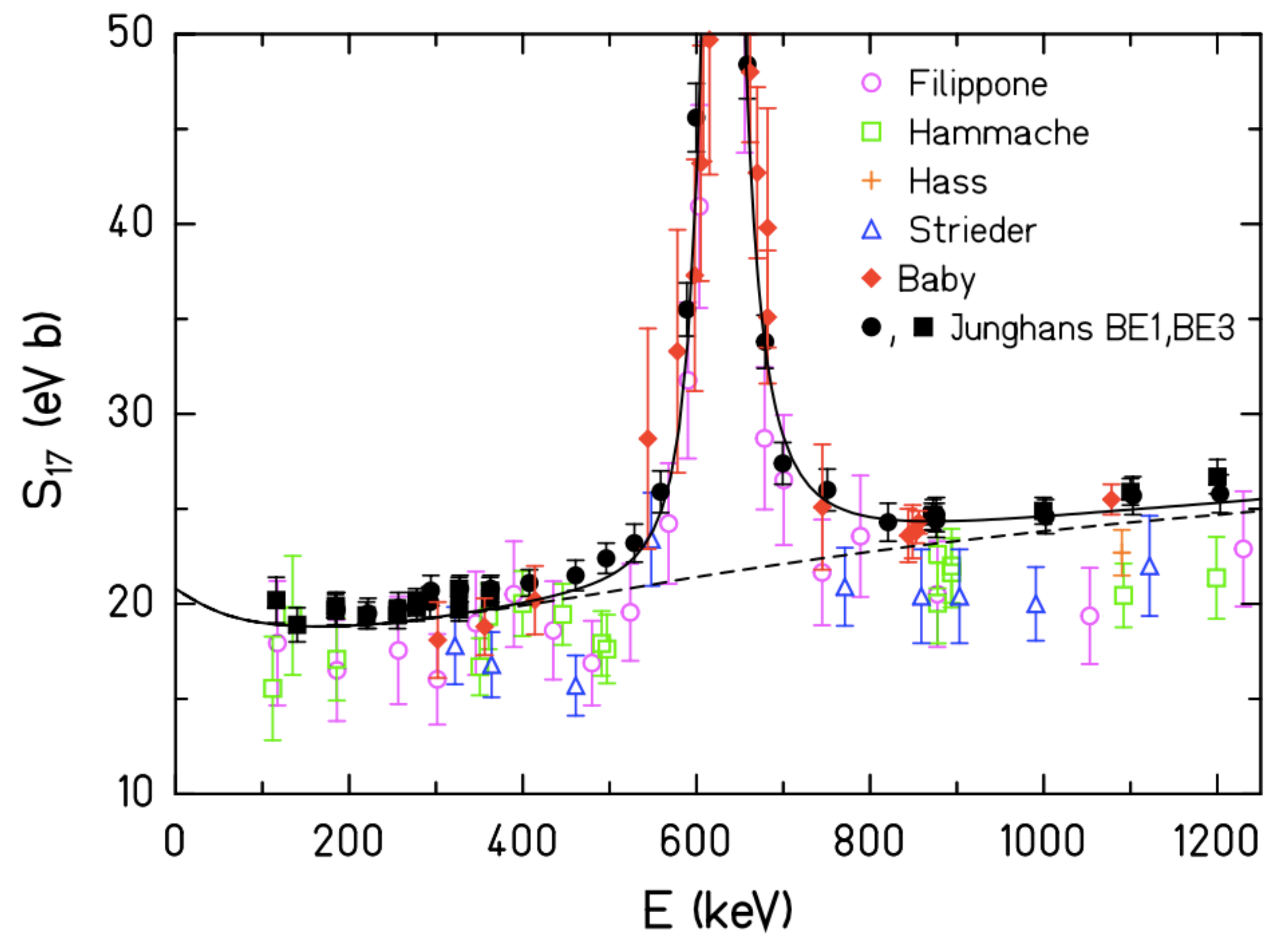
- E1 capture: ^{7}Be + p→^{8}B + γ
- Energies of relevance 20 keV



Status as in “Solar Fusion II”

- Energies of relevance≈20 keV
- There dominated by ^{7}Be-p separations ~10s of fm
- Below narrow 1^+ resonance proceeds via s- and d-wave direct capture
- Energy dependence due to interplay of bound-state properties, Coulomb, strong ISI



- SF II central value used energy-dependence from Descouvemont’s ab initio eight-body calculation. Errors from consideration of energy-dependence in a variety of “reasonable models”

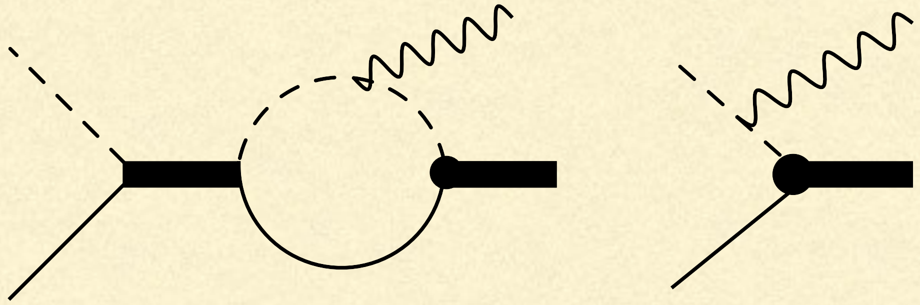
Capture to p-wave halo in EFT

Hammer & DP, NPA (2011)

- At LO: p-wave 1n halo described solely by its ANC and binding energy

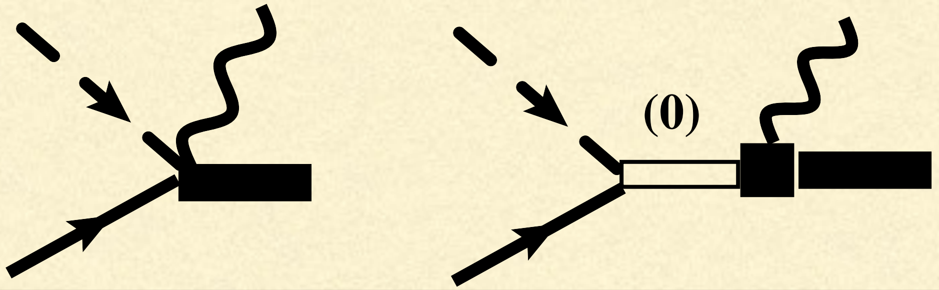
$$u_1(r) = A_1 \exp(-\gamma_1 r)\left(1 + \frac{1}{\gamma_1 r}\right)$$

- Capture to the p-wave state proceeds via the one-body E1 operator: "external direct capture"



$$\mathrm{E1} \propto \int_0^\infty dr\, u_0(r) r u_1(r); \quad u_0(r) = 1 - \frac{r}{a}$$

- NLO: piece of the amplitude representing capture at short distances, represented by a contact operator⇒there is an LEC that must be fit



NLO for ^{7}Be(p,γ)

Zhang, Nollett, Phillips, PRC (2014)
cf. Ryberg, Forssen, Hammer, Platter, EPJA (2014)
Zhang, Nollett, Phillips, PLB (2015); PRC (2018)

- LO calculation: ISI in S=2 & S=1 into p-wave bound state. Scattering wave functions are linear combinations of Coulomb wave functions F_0 and G_0. Bound state wave function=the appropriate Whittaker function

- We also incorporate a low-lying excited state ($1/2^-$) in ^{7}Be

- NLO: piece of the amplitude representing capture at short distances, represented by a contact operator⇒there is an LEC that must be fit

$$S(E) = \quad f(E) \sum_s C_s^2 \left[\left| \mathcal{S}_{\rm EC}\left(E;\delta_s(E)\right) + \overline{L}_s \mathcal{S}_{\rm SD}\left(E;\delta_s(E)\right) + \epsilon_s \mathcal{S}_{\rm CX}\left(E;\delta_s(E)\right) \right|^2 + |\mathcal{D}(E)|^2 \right]$$

Four parameters at LO;
five more at NLO

- ANCs in $^5P_2$ and $^3P_2$: A_{5P2} and A_{3P2}

- Scattering lengths and effective ranges in both $^5S_2$ and $^3S_1$: a_2, r_2 and a_1, r_1

- Core excitation: determined by ratio of ^{8}B couplings of ^{7}Be*p and ^{7}Be-p states: ϵ_1

- LECs associated with contact interaction, one each for S=1 and S=2: L_1 and L_2

Extrapolation to zero energy

Zhang, Nollett, DP, PLB, 2015; arXiv:1708.04017

$$\mathrm{pr}\left(\bar{F}|D;T;I\right) = \int \mathrm{pr}\left(\vec{g},\{\xi_i\}|D;T;I\right)\delta(\bar{F} - F(\vec{g}))d\xi_1\ldots d\xi_5 d\vec{g}$$

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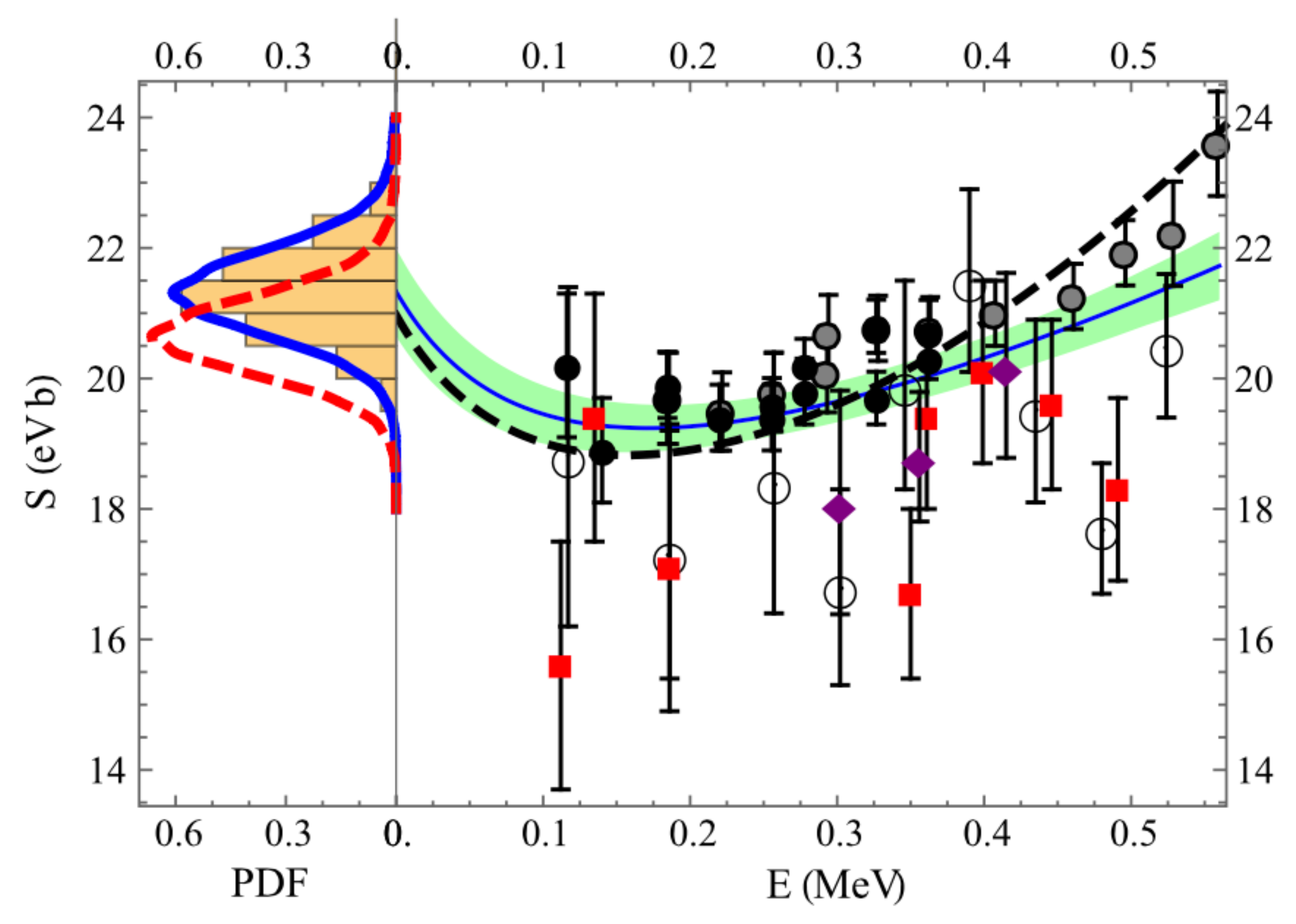

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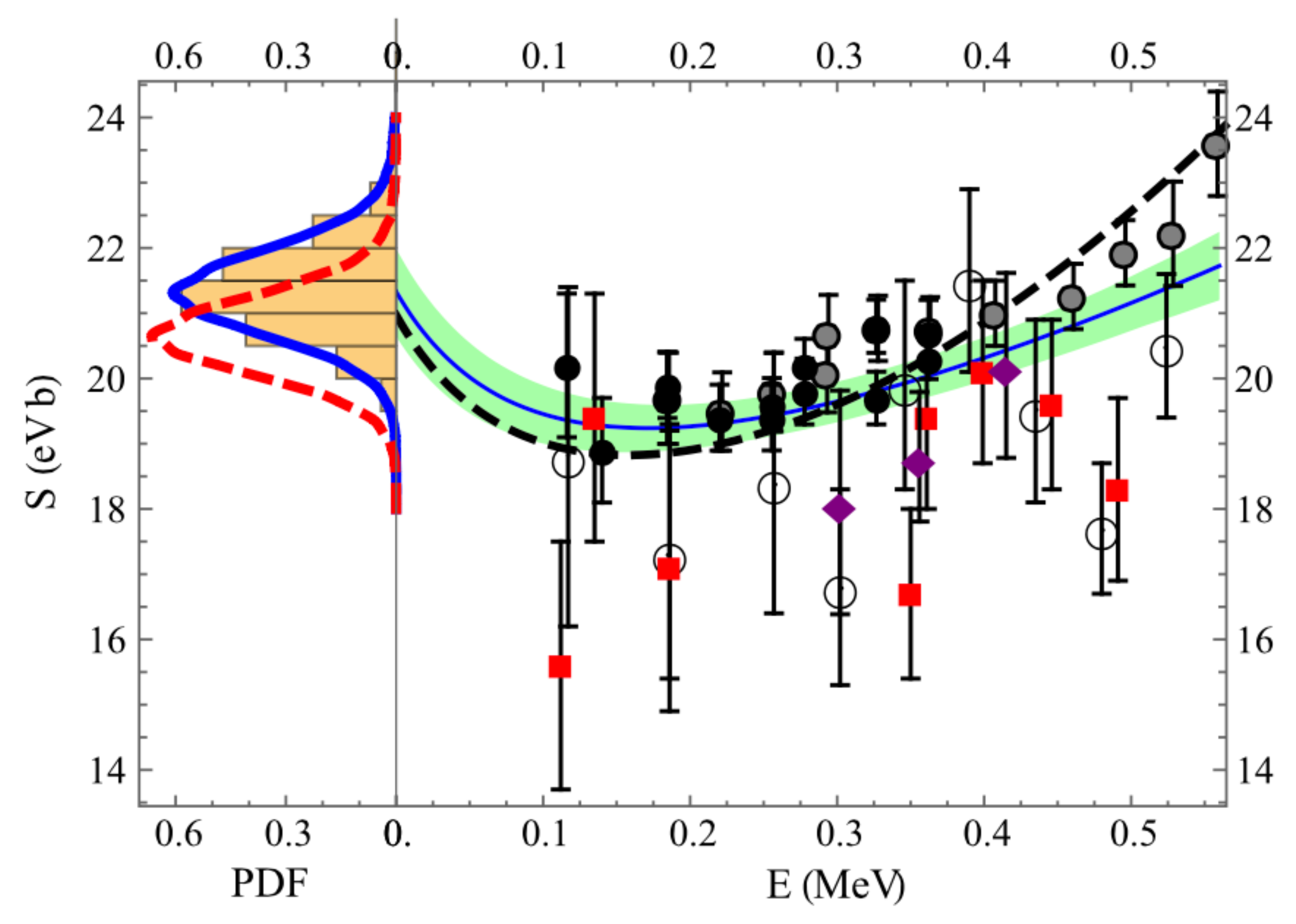

$$S(0) = 21.33^{+0.66}_{-0.69}\ \mathrm{eV\ b}$$

No N^{2}LO corrections

Also assessed impact of N^{3}LO contact operator

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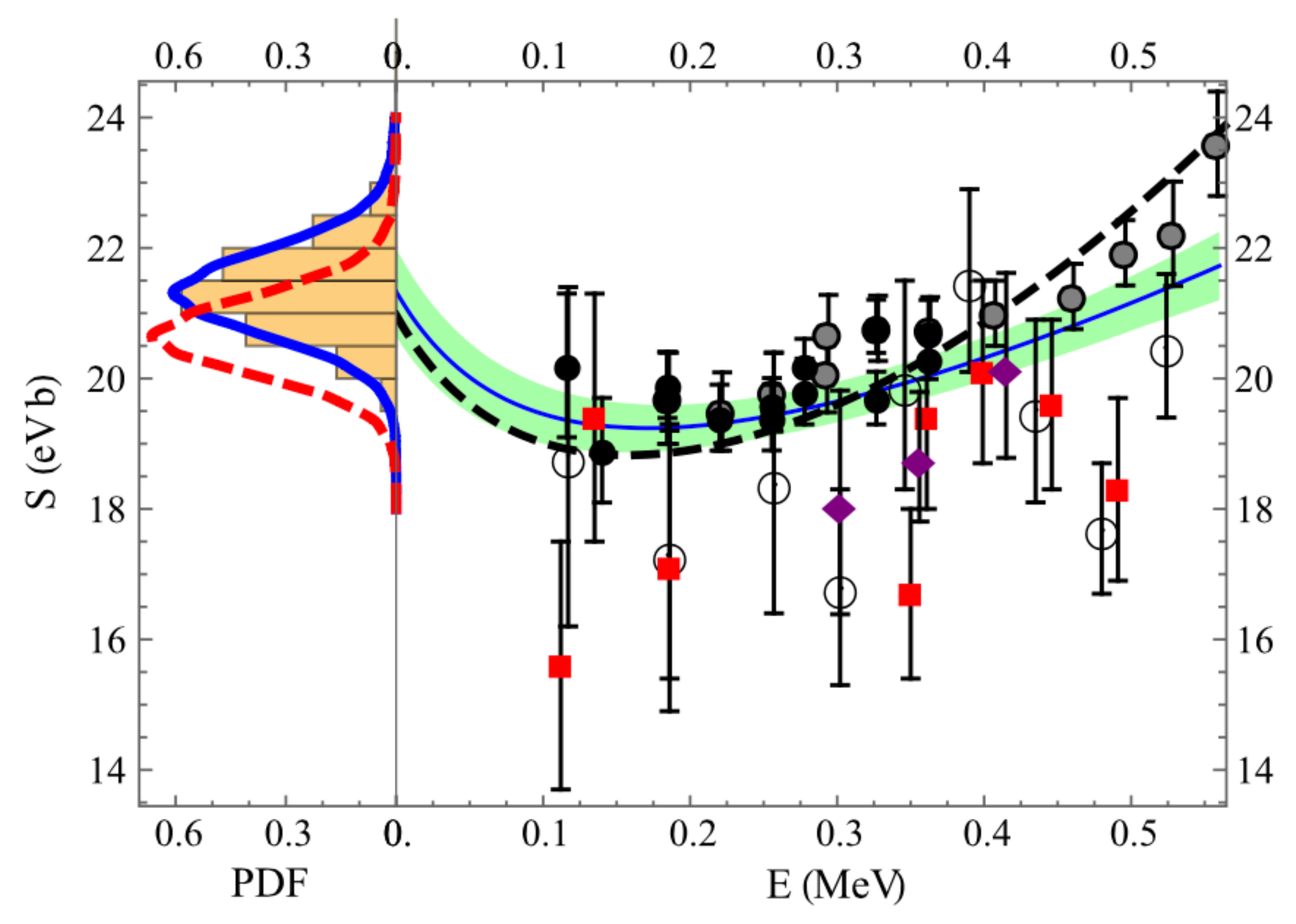

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Some remaining uncertainty due to ^8B S_{1p}

Uncertainty reduced by factor of two: model selection

Ongoing work along these lines

- Simultaneous fit to ^{7}Be+p scattering data: requires inclusion of resonances (TRIUMF experiment)

Poudel, Zhang, DP

- Same techniques applied to ^{3}He(^{4}He,γ)

Vaghani, Higa, Rupak

Zhang, Nollett, DP

- Coulomb dissociation: better reaction theory and connection to *ab initio* structure

Capel, Hammer, DP

- Rotational states as explicit degrees of freedom

Coello Pérez, Papenbrock
Alnamlah, Coello Perez, DP

- Gaussian process models for EFT truncation errors

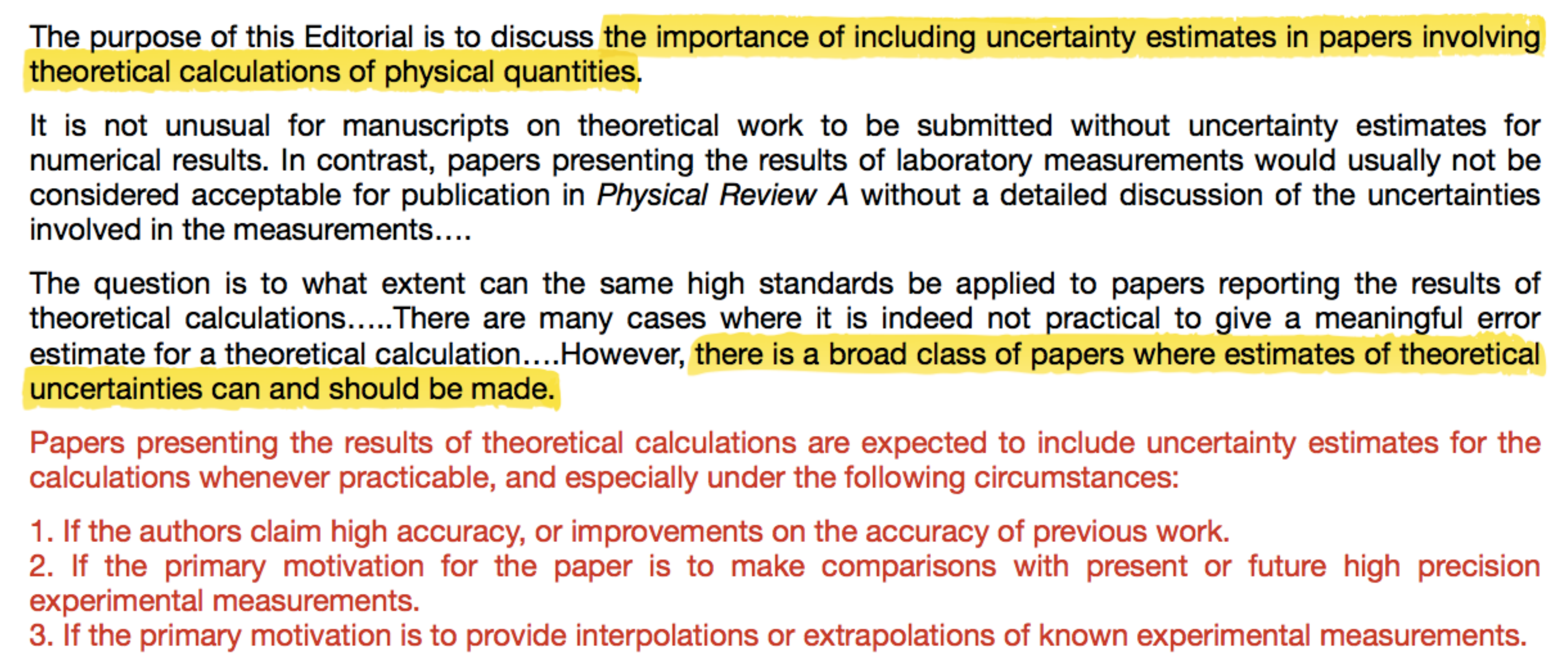
Melendez, Furnstahl, DP, Wesolowski

- χEFT truncation errors in nuclear & neutron matter

Drischler, Melendez, Furnstahl, DP

- Parameter estimation for 3NFs in χEFT

One thing is certain….



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It is not unusual for manuscripts on theoretical work to be submitted without uncertainty estimates for numerical results. In contrast, papers presenting the results of laboratory measurements would usually not be considered acceptable for publication in *Physical Review A* without a detailed discussion of the uncertainties involved in the measurements….

The question is to what extent can the same high standards be applied to papers reporting the results of theoretical calculations…..There are many cases where it is indeed not practical to give a meaningful error estimate for a theoretical calculation….However, there is a broad class of papers where estimates of theoretical uncertainties can and should be made.

Papers presenting the results of theoretical calculations are expected to include uncertainty estimates for the calculations whenever practicable, and especially under the following circumstances:

1. If the authors claim high accuracy, or improvements on the accuracy of previous work.
2. If the primary motivation for the paper is to make comparisons with present or future high precision experimental measurements.
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Physical Review A Editorial, 29 April 2011

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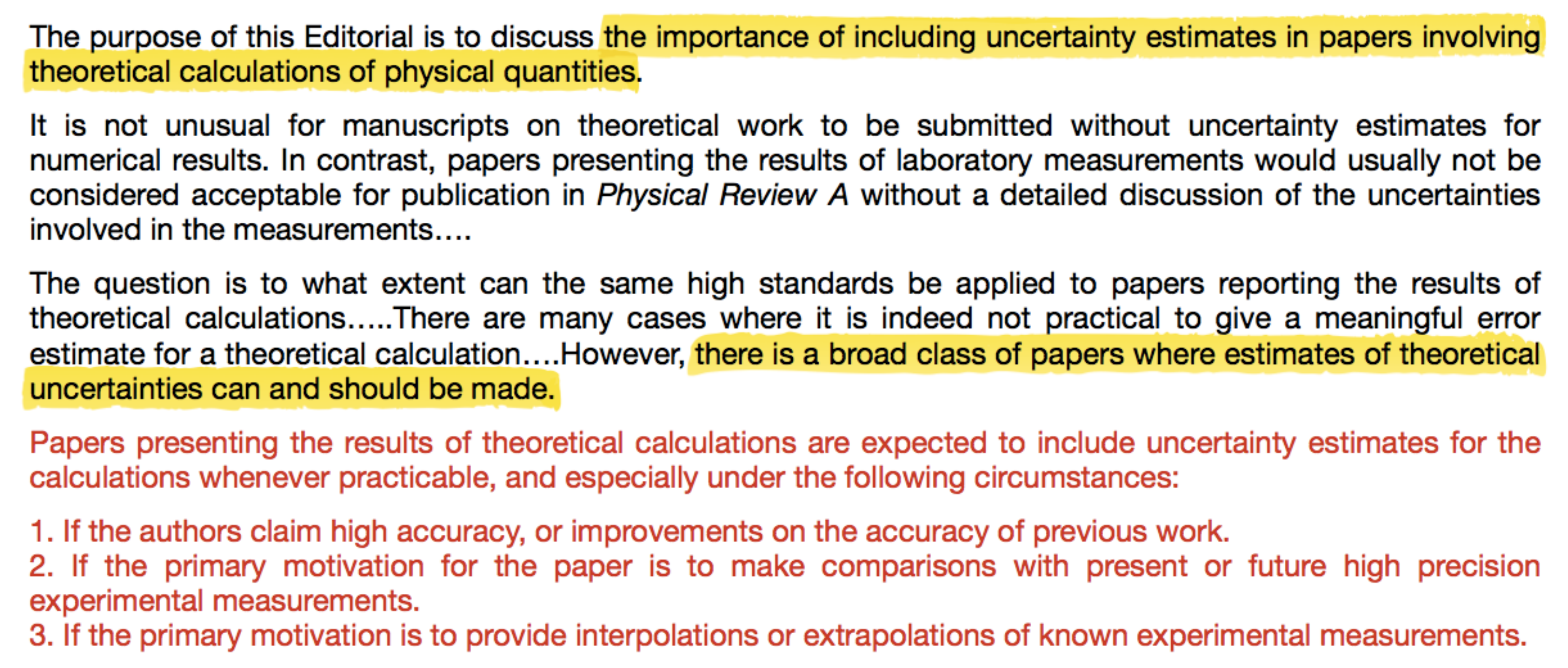

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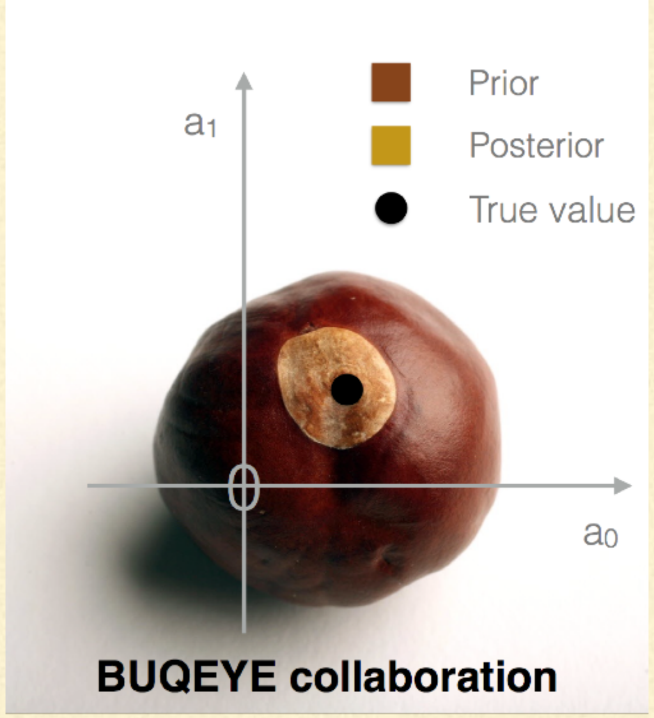

Bayesian Uncertainty Quantification:
Errors for Your EFT

Theorists Anonymous

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- Make amends for those mistakes
- Help others who must deal with the same issues

References

- “A recipe for EFT uncertainty quantification in nuclear physics”, R. J, Furnstahl, D. R. Phillips, S. Wesolowski, J. Phys, G **42**, 034028 (2014).
- “Quantifying truncation errors in effective field theory”, R. J, Furnstahl, N. Klco, D. R. Phillips, S. Wesolowski, Phys. Rev. C **92**, 024005 (2015); “Bayesian truncation errors in chiral EFT: nucleon-nucleon observables”, J. Melendez, S. Wesolowski, R. J. Furnstahl, Phys. Rev. C **96**, 024003 (2017).
- “Bayesian parameter estimation for effective field theories”, S. Wesolowski, N. Klco, R. J. Furnstahl, D. R. Phillips, A. Thapaliya, J. Phys G **43**, 074001 (2016); “Exploring Bayesian Parameter Estimation for ChiEFT using NN phase shifts”, S. Wesolowski, R. J. Furnstahl, J. Melendez, D. R. Phillips, arXiv:1808.08211
- “Halo effective field theory constrains the solar ^{7}Be + p→^{8}B + γ rate”, X. Zhang, K. Nollett, D. R. Phillips, Phys. Lett. **B751**, 535 (2015); “Models, measurement and EFT: proton capture on ^{7}Be at NLO”, Phys. Rev. C **98**, 034616 (2018).
- “Effective field theory for halo nuclei”, H.-W. Hammer, C. Ji, D. R. Phillips, Topical Review for J. Phys. G **44**, 103002 (2017).

Backup Slides

A Generic EFT

$$g(x) = \sum_{i=0}^{k} \mathcal{A}_i(x) x^i$$

$$x = \frac{p}{\Lambda_b}$$

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- Suppose we are interested in a quantity as a function of a momentum, p, that is small compared to some high scale, Λ_b.
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- $f_i(x, \mu)$ is a calculable function, that encodes IR physics at order i

- a_i is a low-energy constant (LEC): encodes UV physics at order i. Must be fit to data

- Complications: multiple light scales, multiple functions at a given order, skipped orders, ….

Bayes→Result

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- Bayes theorem: $\mathrm{pr}(\bar{c}|c_0, c_1, \ldots, c_k) = \dfrac{\mathrm{pr}(c_0, c_1, \ldots, c_k|\bar{c})\mathrm{pr}(\bar{c})}{\mathrm{pr}(c_0, c_1, \ldots, c_k)}$

$$= \mathcal{N}\mathrm{pr}(\bar{c})\Pi_{n=0}^{k}\mathrm{pr}(c_n|\bar{c})$$

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- Marginalization:

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- This is generic, but the integrals are simple in the case of "Prior A"

$$\mathrm{pr}(\bar{c}|c_0, c_1, \ldots, c_k) \propto \begin{cases} 0 & \text{if } \bar{c} < \max\{c_0, \ldots, c_k\} \\ 1/\bar{c}^{k+2} & \text{if } \bar{c} > \max\{c_0, \ldots, c_k\} \end{cases}$$

$$\mathrm{pr}(c_{k+1}|c_0, c_1, \ldots, c_k) \propto \begin{cases} 1 & \text{if } c_{k+1} < c_{\max} \\ \left(\frac{c_{\max}}{c_{k+1}}\right)^{k+2} & \text{if } c_{k+1} > c_{\max} \end{cases}$$

I don't like THAT prior!

- Modify Set A to restrict cbar to a finite range, e.g. $A_{[0.25,4]}$
- Set B: give cbar a log-normal prior: $\mathrm{pr}(\bar{c}) = \frac{1}{\sqrt{2\pi}\bar{c}\sigma} e^{-(\log \bar{c})^2/2\sigma^2}$
- Set C: $\mathrm{pr}(c_n|\bar{c}) = \frac{1}{\sqrt{2\pi}\bar{c}} e^{-c_n^2/2\bar{c}^2}$; $\mathrm{pr}(\bar{c}) \propto \frac{1}{\bar{c}}\theta(\bar{c} - \bar{c}_<)\theta(\bar{c}_> - \bar{c})$
- Same formulas as before can be invoked. Now numerical.

$$\mathrm{pr}(c_{k+1}|c_0, c_1, \ldots, c_k) = \int_0^\infty d\bar{c}\, \mathrm{pr}(c_{k+1}|\bar{c})\mathrm{pr}(\bar{c}|c_0, c_1, \ldots c_k)$$

$$\mathrm{pr}(\bar{c}|c_0, c_1, \ldots, c_k) = \mathcal{N}\mathrm{pr}(\bar{c})\Pi_{n=0}^{k}\mathrm{pr}(c_n|\bar{c})$$

- You don't like these? Pick your own and follow the rules…
- First omitted term approximation

BREAKDOWN-SCALE INFERENCE

- Λ_b determines the size of the c_n's. Choose it too big, and they'll be too big. Choose it too small, they'll be too small. And progressively so as one moves to higher and higher order.

- We have a theory for $\mathrm{pr}(c_n|c_0, c_1, \ldots, c_k)$: now use Bayes' theorem to see how (im)probable are the c_n's that dimensionful EFT coefficients (b_n's) produce for a given Λ_b.

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(NLO: k=2, NNLO: k=3, N^3LO: k=4, etc.)

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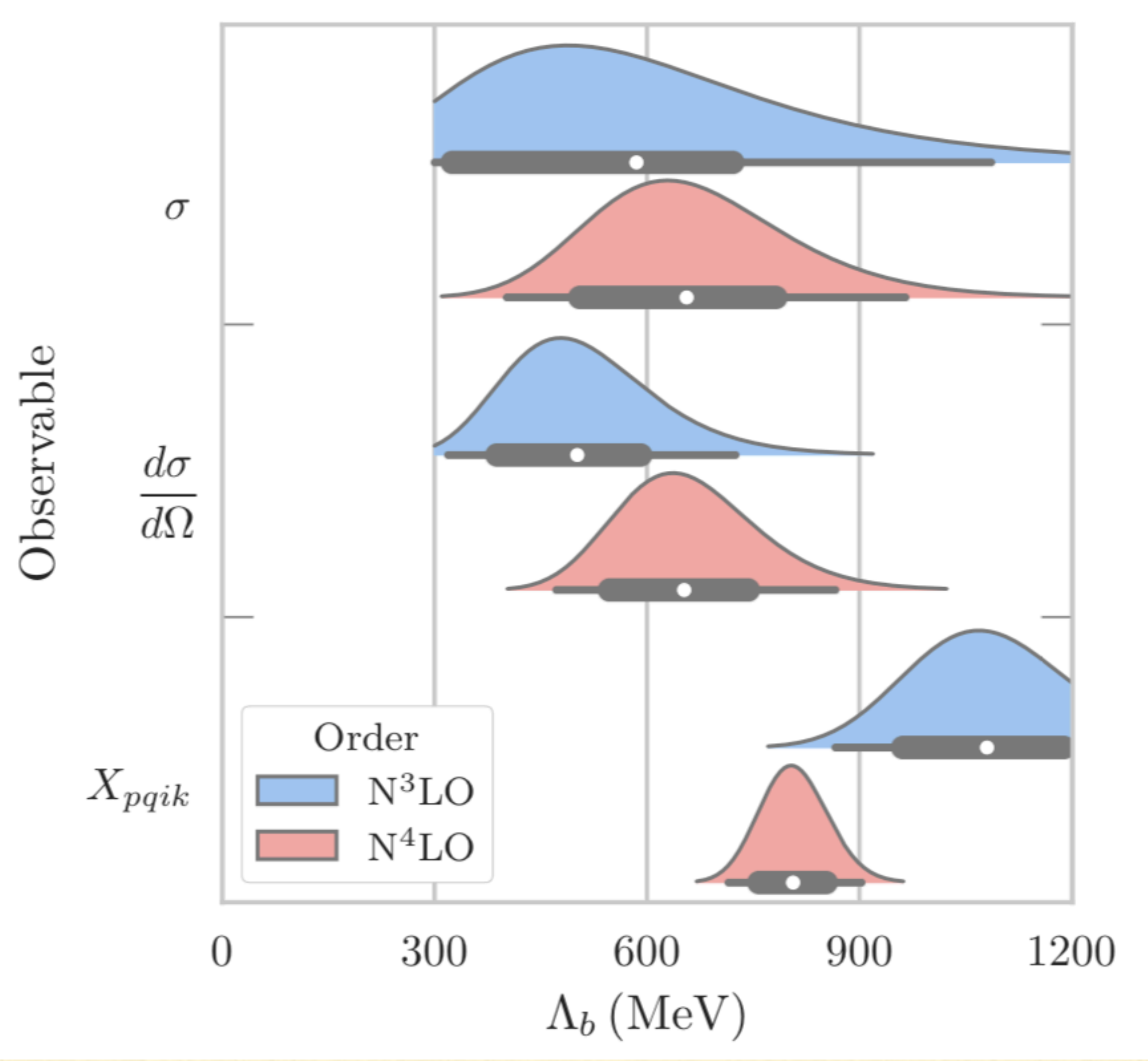
Using 5 energies (and 2 angles):



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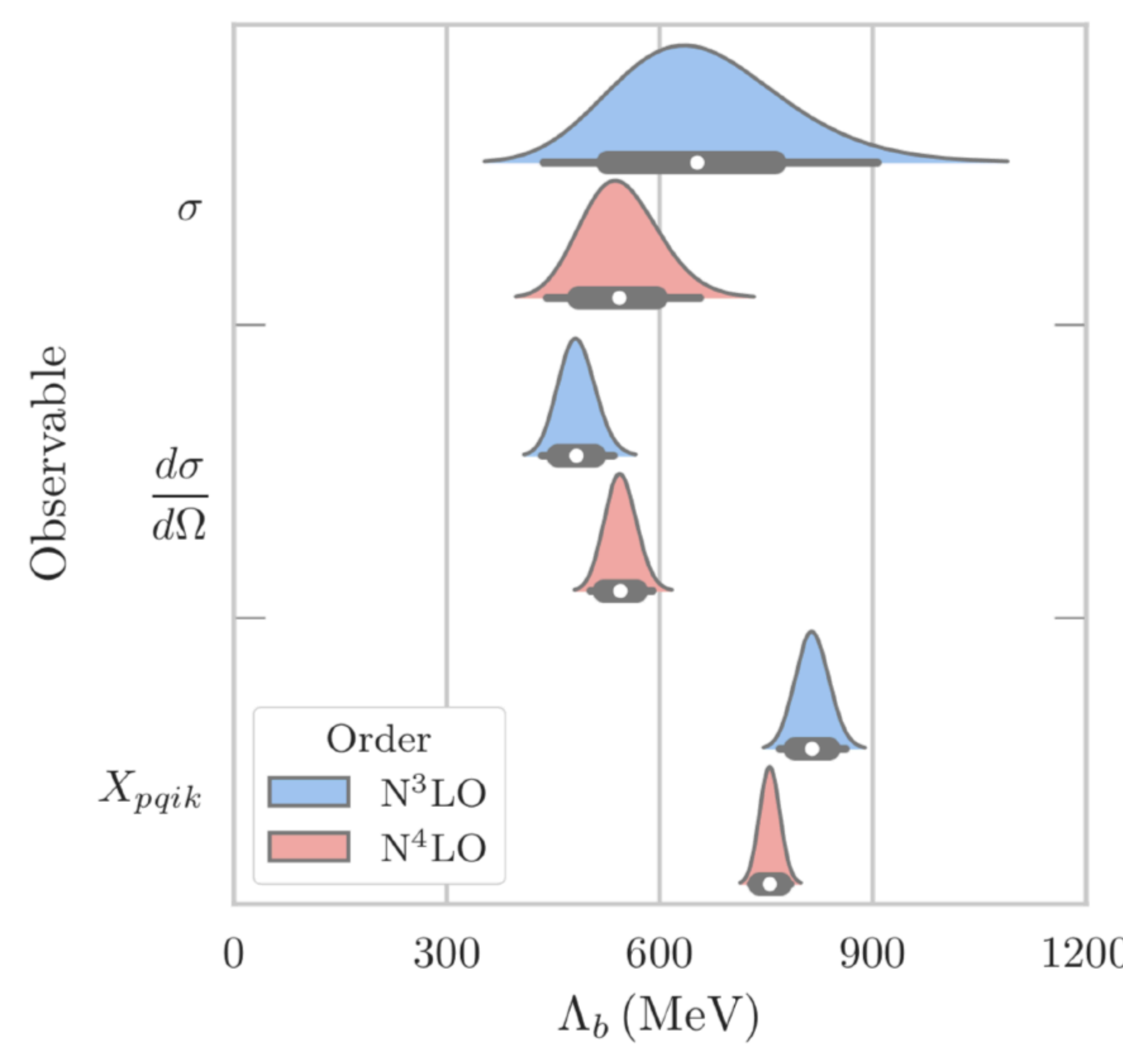
Using 17 energies (and 7 angles):



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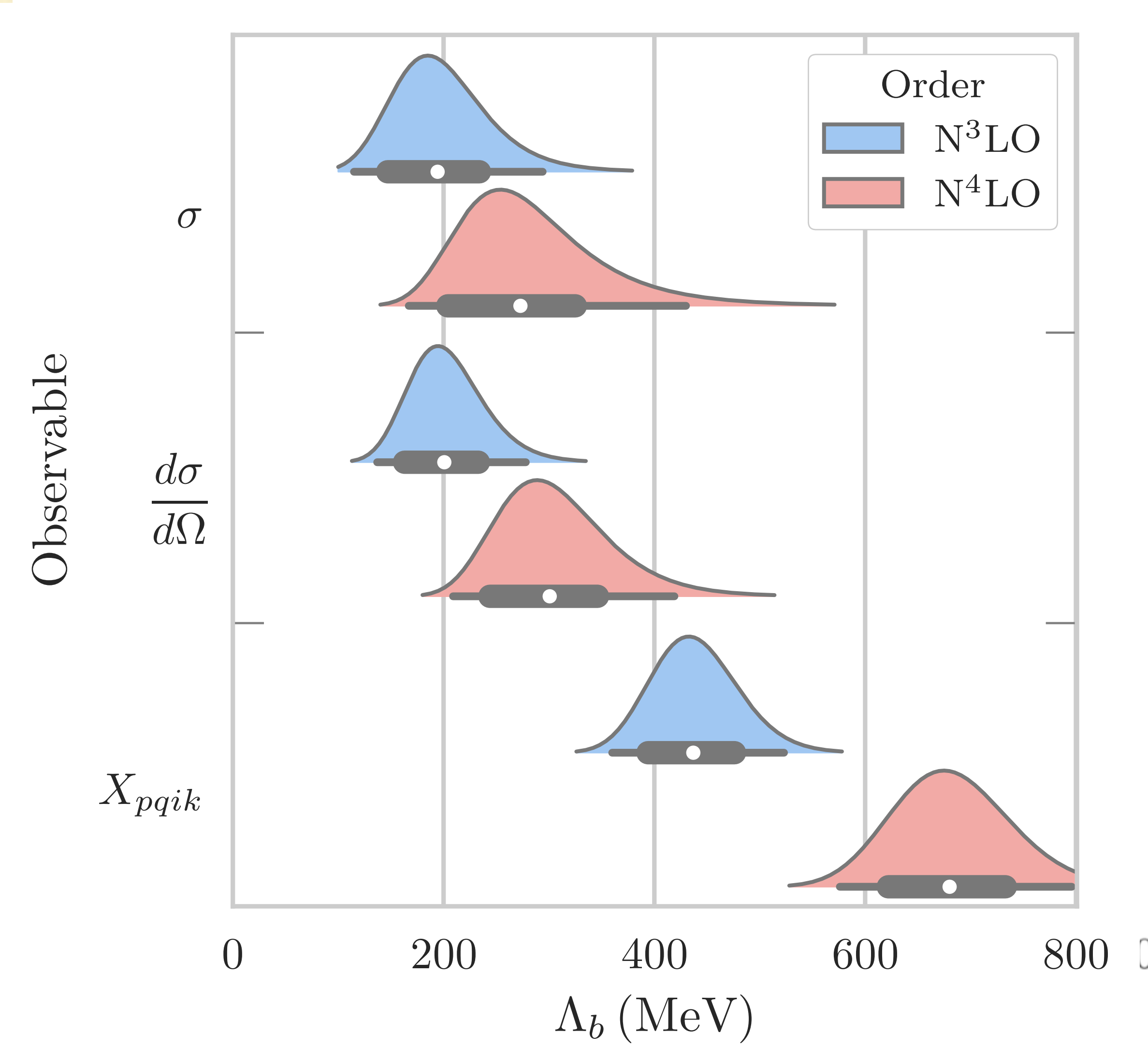
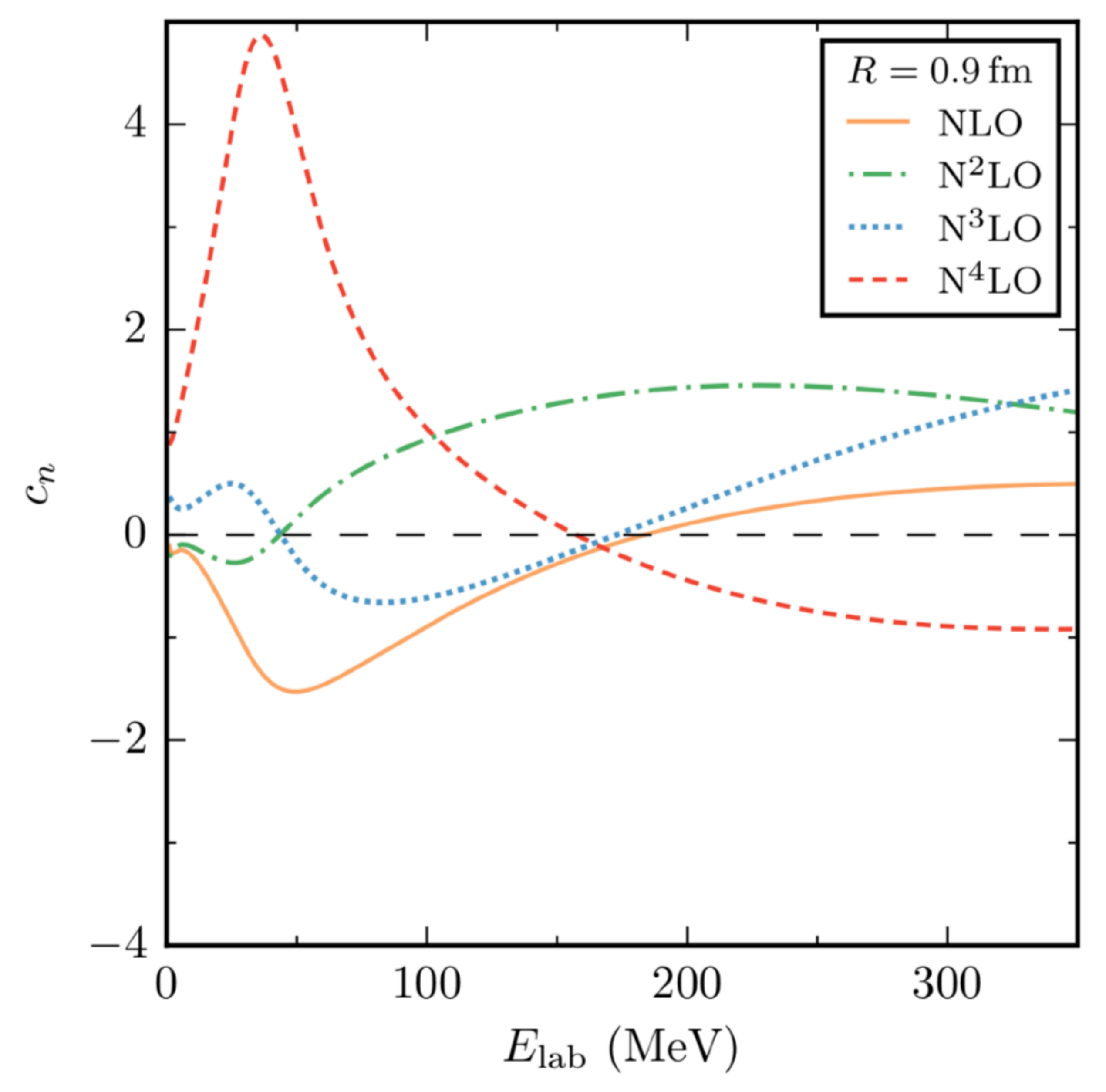
R=1.2 fm



FUNCTIONAL DATA

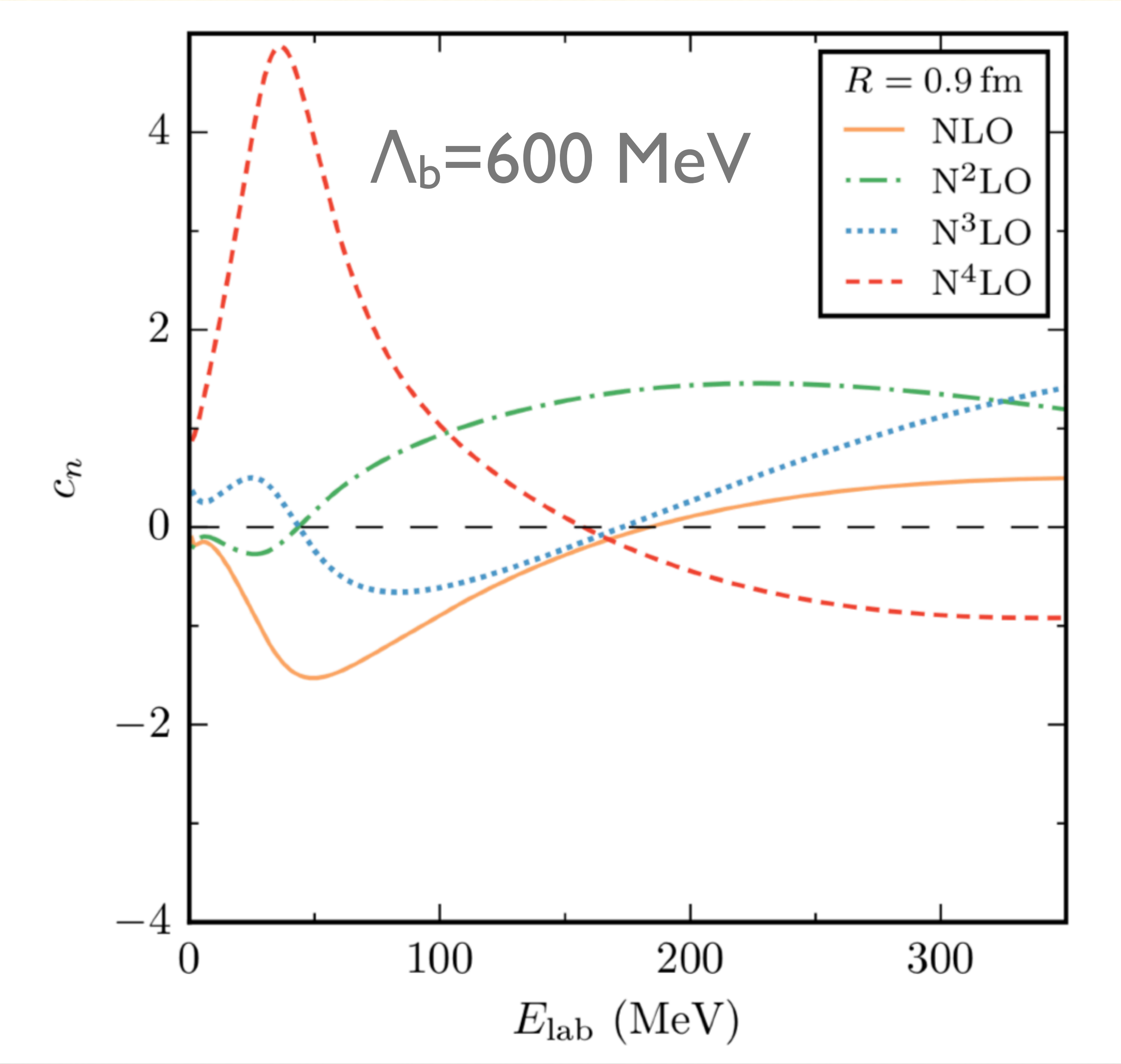
- But we don't have 119 independent data points
- We have a function for each observable at each order
- Can we understand the properties of these functions, so we can do Λ_b inference and compute success ratios rigorously?



$$\sigma(E) = \sigma_0(E)\left[1 + c_2(E)x^2 + c_3(E)x^3 + c_4(E)x^4 + c_5(E)x^5\right]$$

OBSERVATIONS AND QUESTIONS

- c_n's do not grow or shrink with n: good Λ_b choice
- Bounded functions, mostly between -2 and 2
- Each "takes a turn" at being largest
- Not oscillating quickly in this energy range



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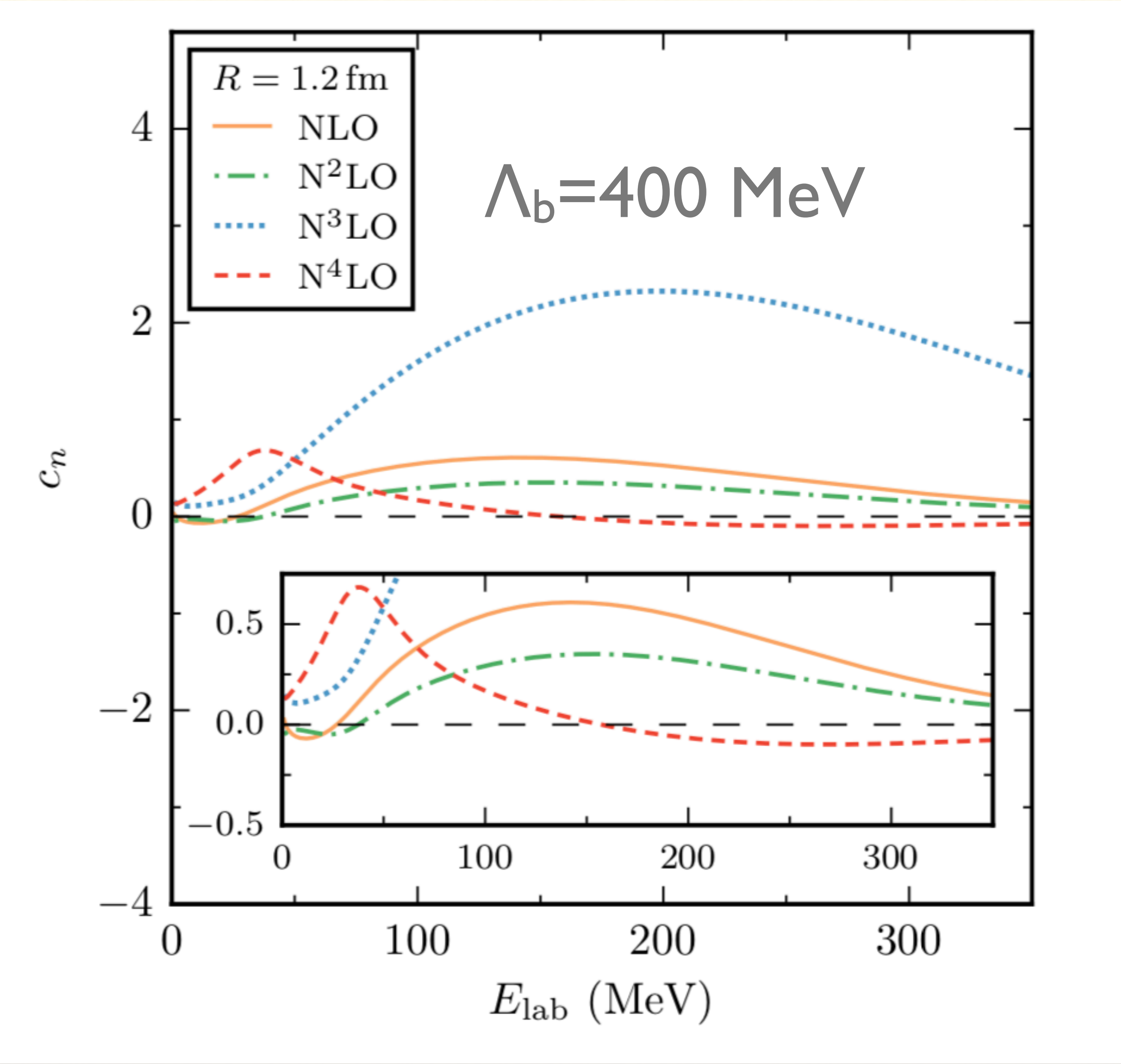

OBSERVATIONS AND QUESTIONS

- c_n's do not grow or shrink with n: good Λ_b choice
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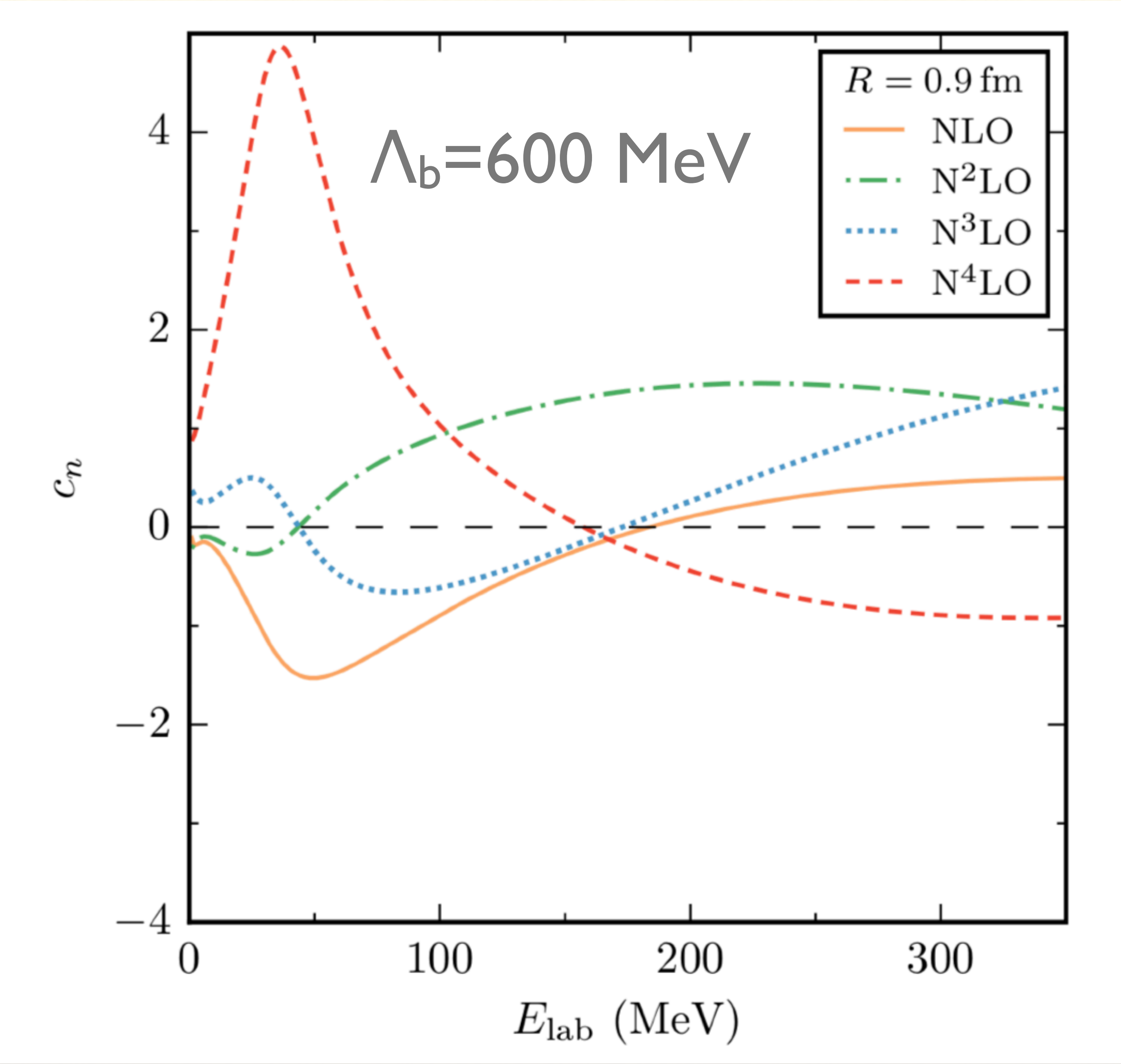

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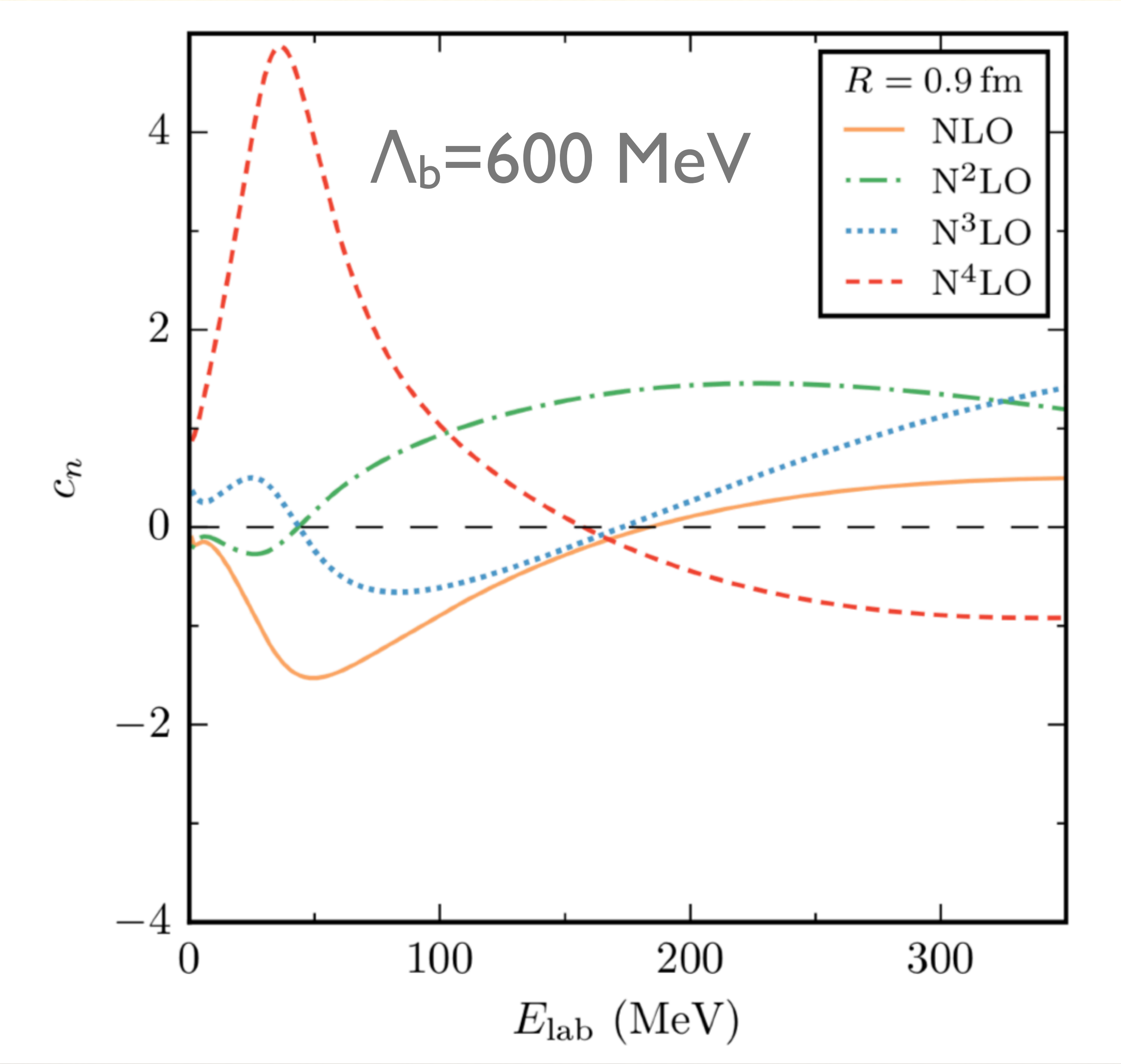


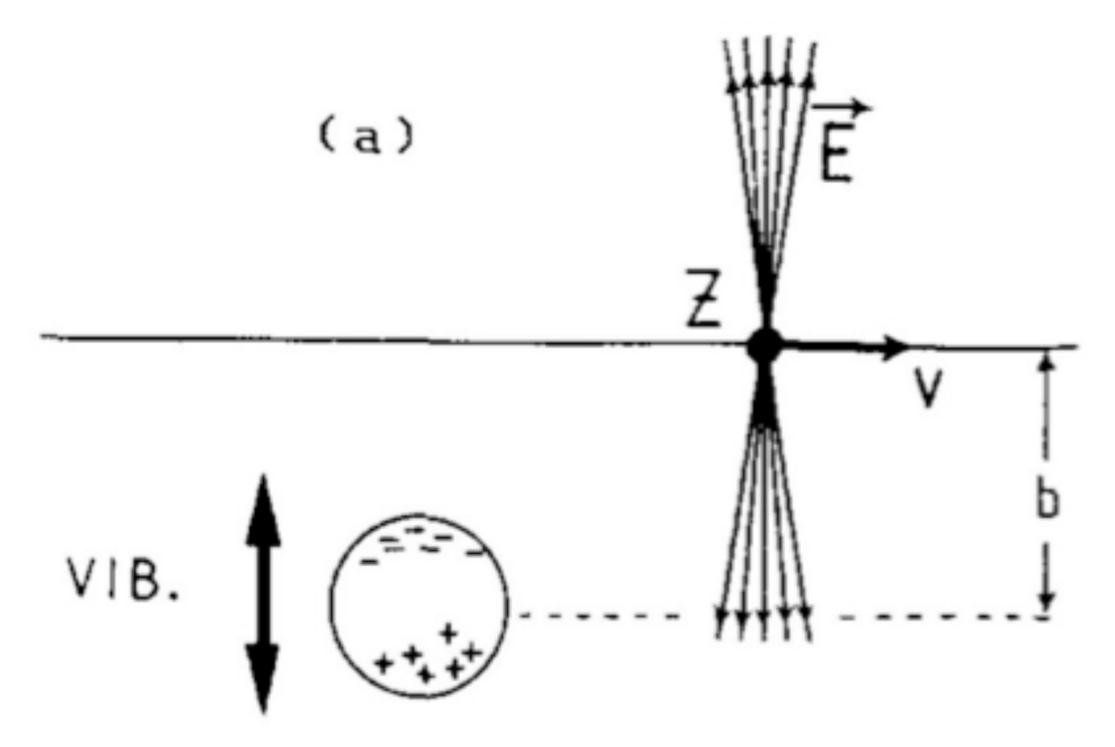
Physics questions:

- Do curves all fluctuate around zero with some common variance?
- What is the correlation length? Is it different at each order?

Coulomb dissociation of halo nuclei

Bertulani, arXiv:0908.4307

- Coulomb dissociation: collide halo nucleus (we hope peripherally) with a high-Z nucleus
- Do with different Z, different nuclear sizes, different energies to test systematics



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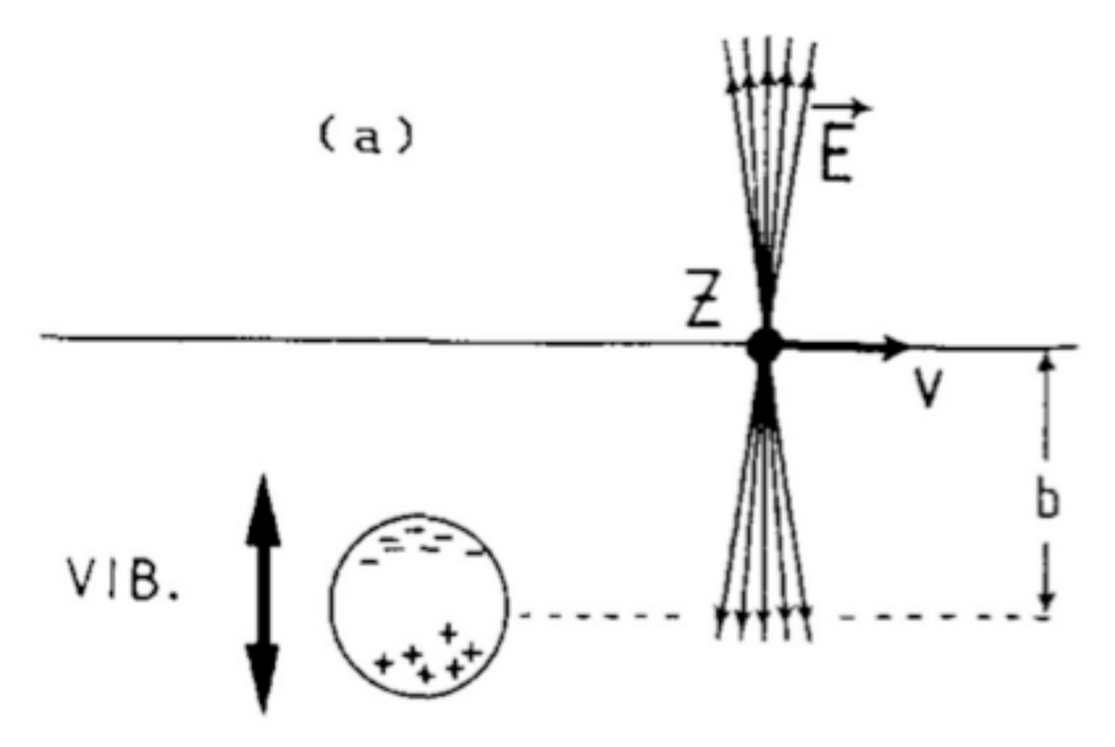

- Coulomb excitation dissociation cross section (p.v. b≫R$_{\text{target}}$)

$$\frac{d\sigma_C}{2\pi b db} = \sum_{\pi L} \int \frac{dE_\gamma}{E_\gamma} n_{\pi L}(E_\gamma, b) \sigma_\gamma^{\pi L}(E_\gamma)$$

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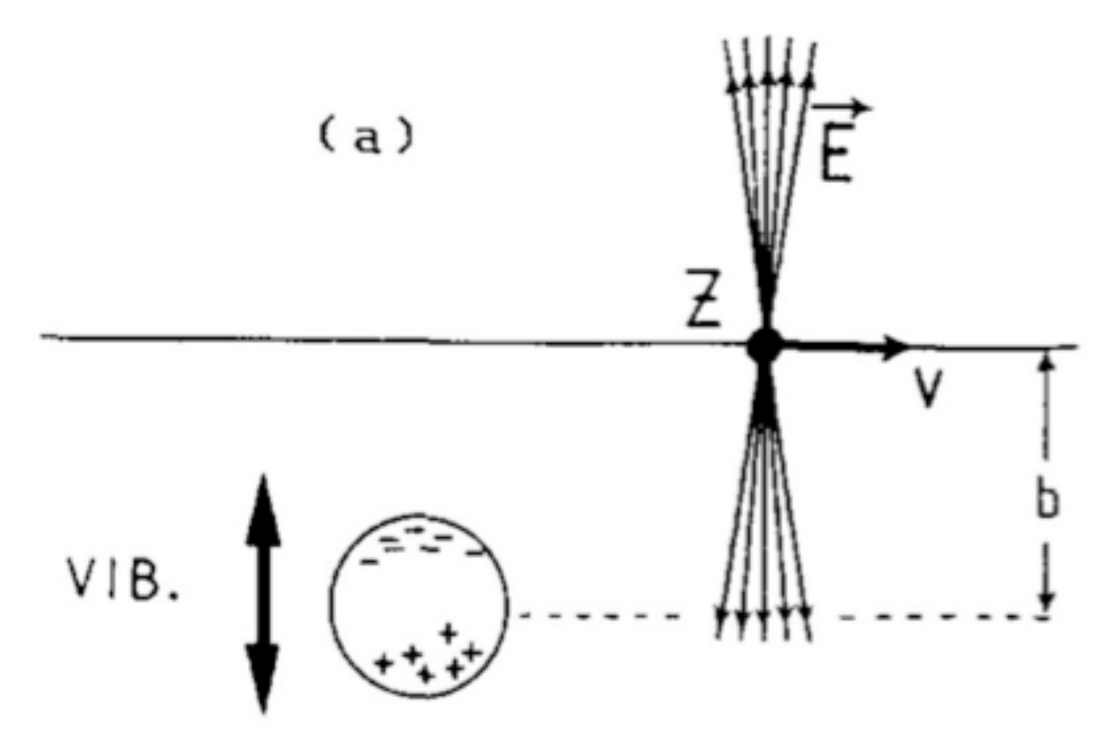

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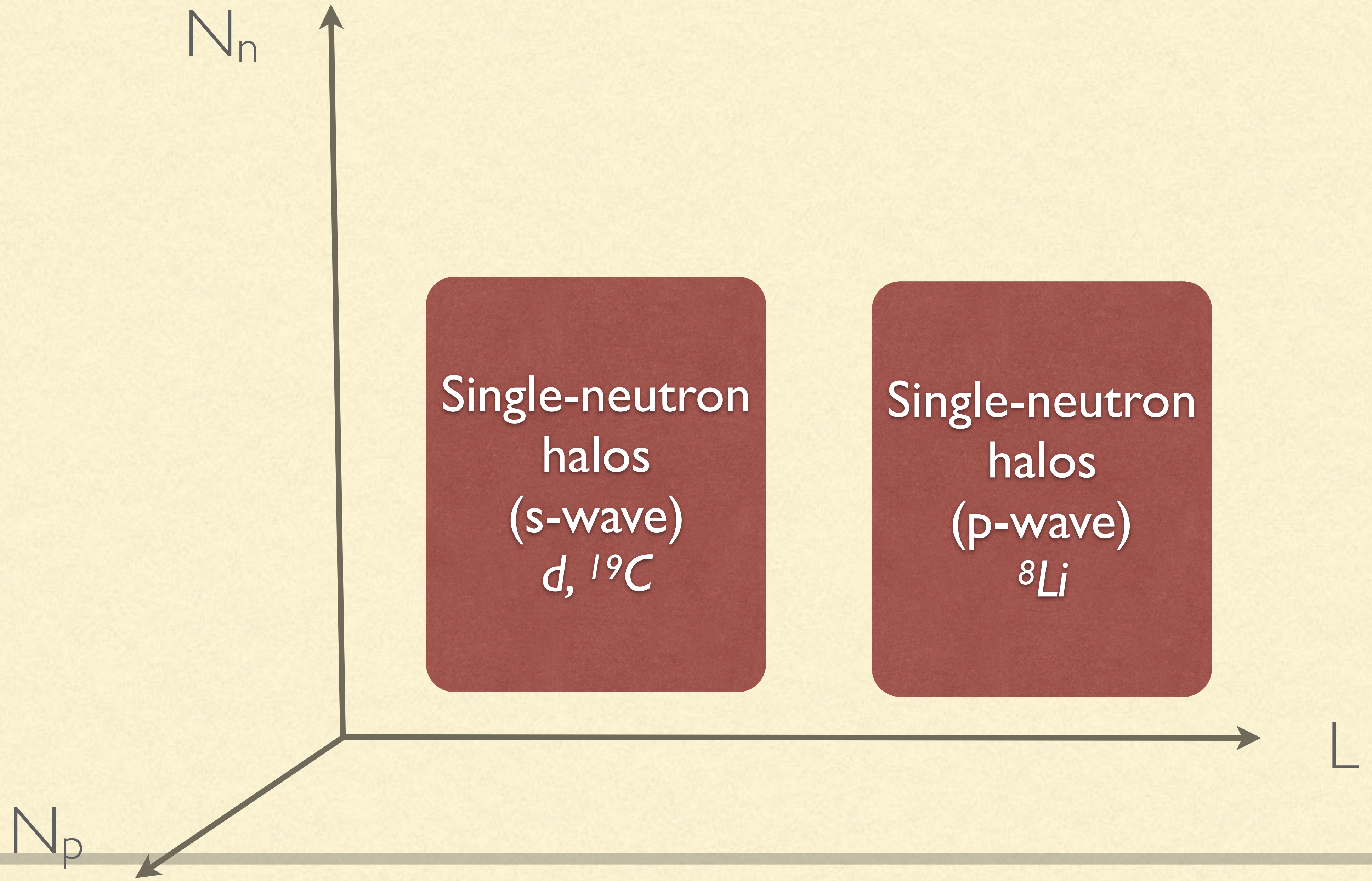
- $\sigma_\gamma^{\pi L}(E_\gamma)$ can then be extracted: it's the (total) cross section for dissociation of the nucleus due to the impact of photons of multipolarity πL.

The multi-dimensional Halo EFT space



Single-neutron halos (s-wave)
d, ^{19}C

Single-neutron halos (p-wave)
^{8}Li

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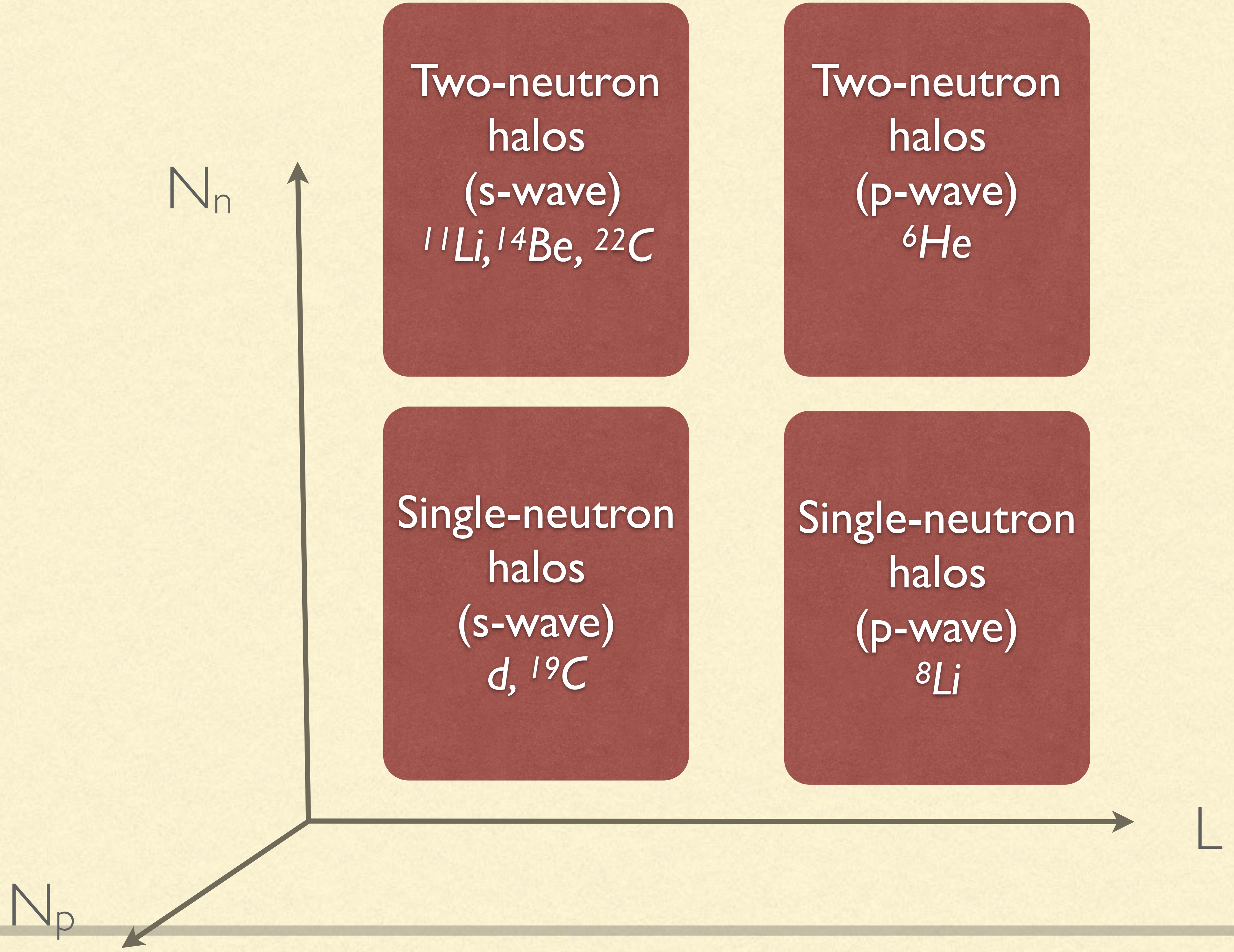

N_n

Two-neutron halos (s-wave)
^{11}Li, ^{14}Be, ^{22}C

Two-neutron halos (p-wave)
^{6}He

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L

N_p

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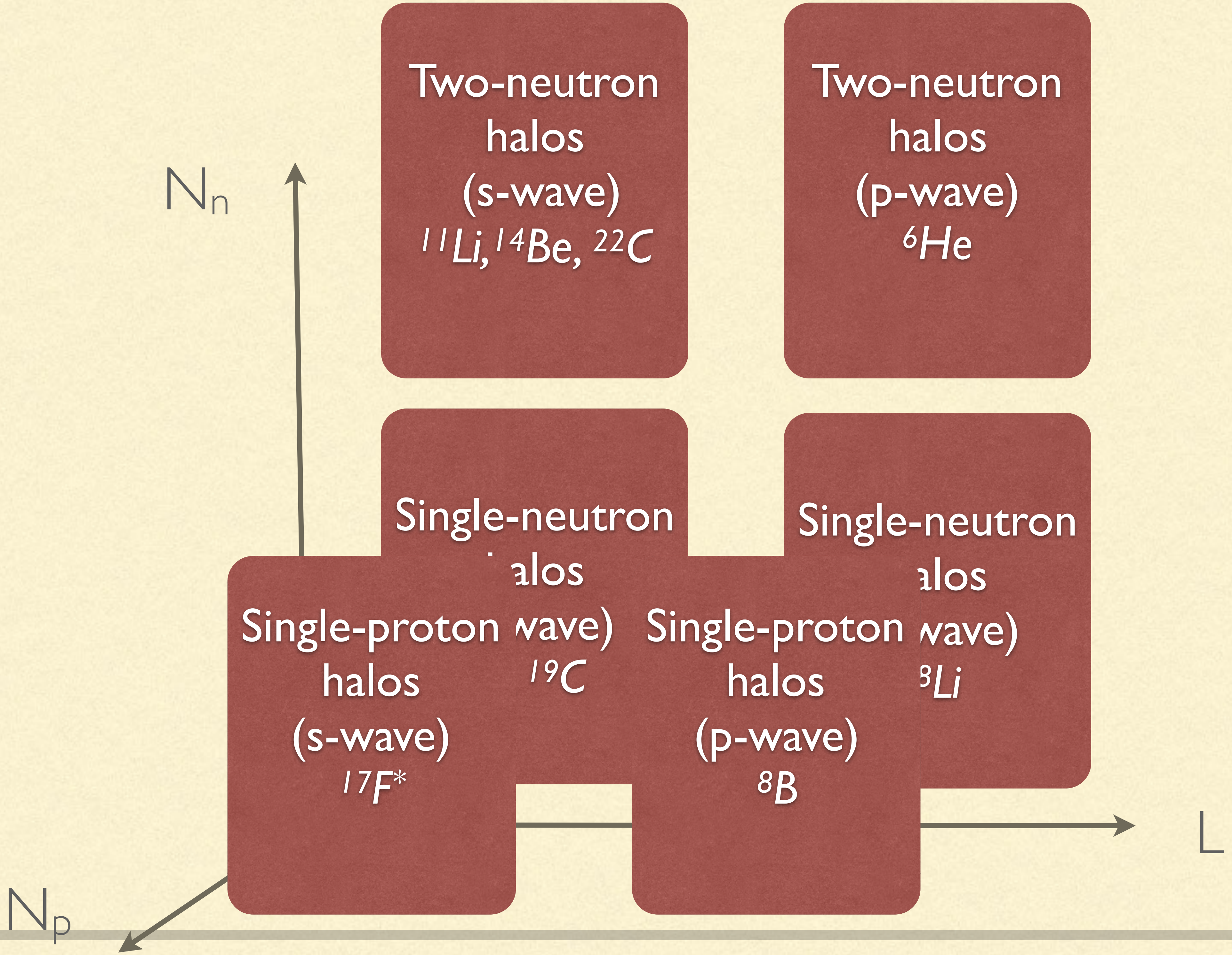

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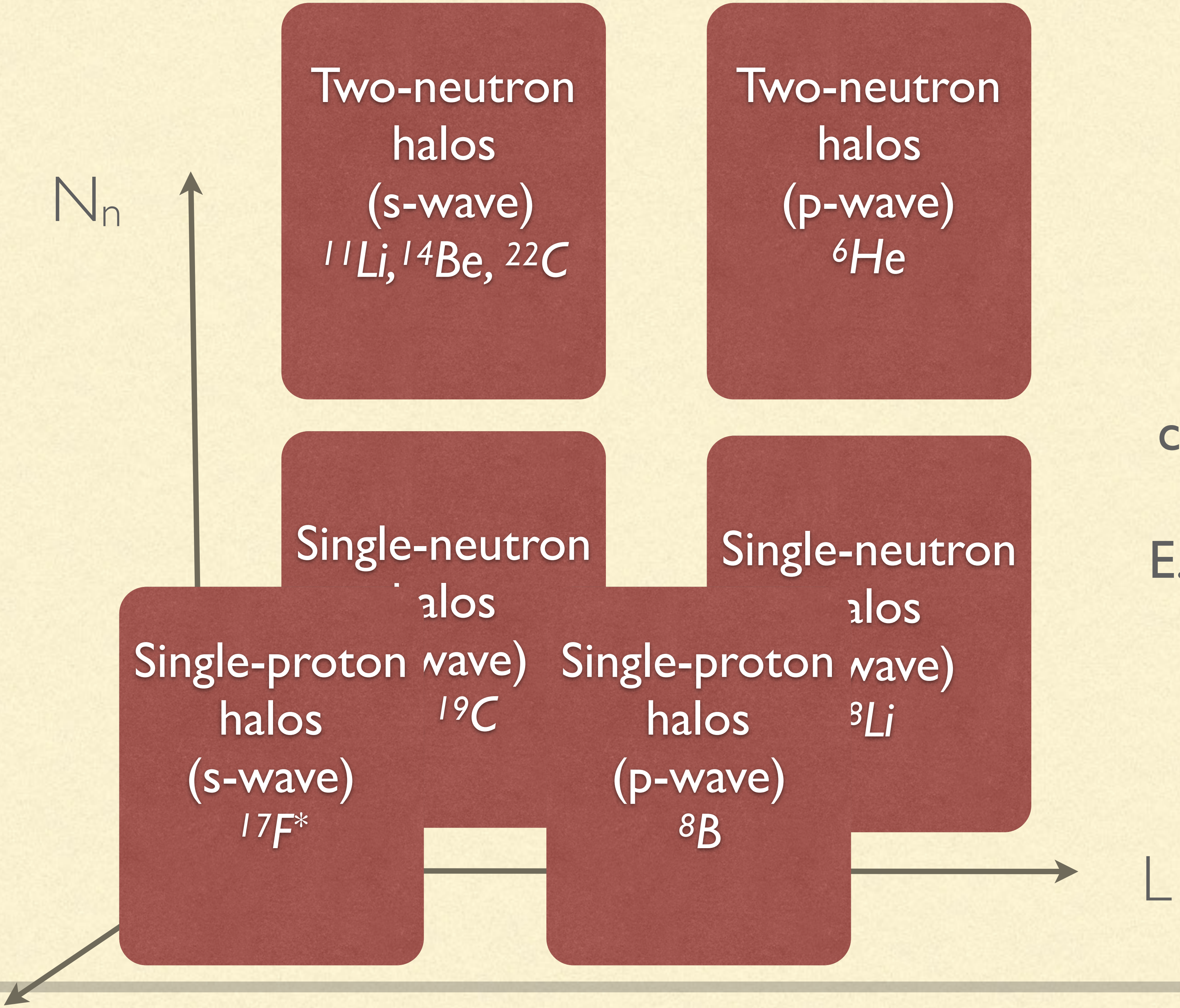
Single-proton halos (s-wave)
*^{17}F**

Single-proton halos (p-wave)
^{8}B

The multi-dimensional Halo EFT space



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Plus
complementary
direction: N_α
E.g. *^{9}Be*, *^{12}C**, etc.

Lagrangian for s- and p-wave states

s-wave: Kaplan, Savage, Wise (1998); van Kolck (1999); Birse, Richardosn, McGovern 1999)

p-wave: Bertulani, Hammer, van Kolck (2002); Bedaque, Hammer, van Kolck (2003)

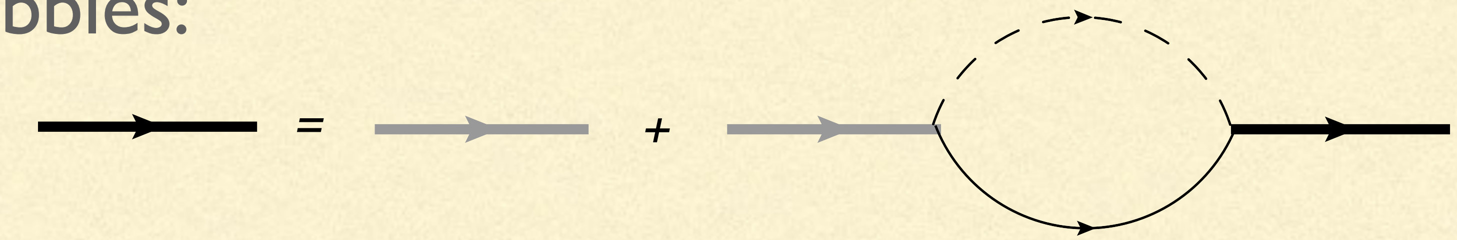
$$\begin{aligned}\mathcal{L} \quad = \quad & c^\dagger\left(i\partial_t + \frac{\nabla^2}{2M}\right)c + n^\dagger\left(i\partial_t + \frac{\nabla^2}{2m}\right)n \\ & +\sigma^\dagger\left[\eta_0\left(i\partial_t + \frac{\nabla^2}{2M_{nc}}\right) + \Delta_0\right]\sigma + \pi_j^\dagger\left[\eta_1\left(i\partial_t + \frac{\nabla^2}{2M_{nc}}\right) + \Delta_1\right]\pi_j \\ & -g_0\left[\sigma n^\dagger c^\dagger + \sigma^\dagger n c\right] - \frac{g_1}{2}\left[\pi_j^\dagger(n\; i\overleftrightarrow{\nabla}_j\; c) + (c^\dagger\; i\overleftrightarrow{\nabla}_j\; n^\dagger)\pi_j\right] \\ & -\frac{g_1}{2}\frac{M-m}{M_{nc}}\left[\pi_j^\dagger\; i\overrightarrow{\nabla}_j\;(nc) - i\overleftrightarrow{\nabla}_j\;(n^\dagger c^\dagger)\pi_j\right] + \ldots\,,\end{aligned}$$

- c, n: "core", "neutron" fields. c: boson, n: fermion
- σ, π_j: S-wave and P-wave fields
- Minimal substitution generates leading EM couplings

Dressing the s-wave state

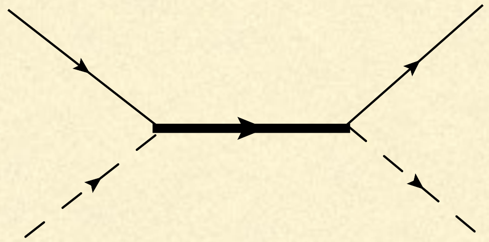
Kaplan, Savage, Wise; van Kolck; Gegelia;
Birse, Richardson, McGovern

- σnc coupling g_0 of order $R_{\rm halo}$, nc loop of order $1/R_{\rm halo}$. Therefore need to sum all bubbles:



$$D_\sigma(p) = \frac{1}{\Delta_0 + \eta_0[p_0 - \mathbf{p}^2/(2M_{nc})] - \Sigma_\sigma(p)}$$

$$\Sigma_\sigma(p) = -\frac{g_0^2 m_R}{2\pi}\left[\mu + i\sqrt{2m_R\left(p_0 - \frac{\mathbf{p}^2}{2M_{nc}} + i\eta\right)}\right] \qquad \text{(PDS)}$$



$$t = \frac{2\pi}{m_R}\frac{1}{\frac{1}{a_0} - \frac{1}{2}r_0k^2 + ik}$$

$$D_\sigma(p) = \frac{2\pi\gamma_0}{m_R^2 g_0^2}\frac{1}{1 - r_0\gamma_0}\frac{1}{p_0 - \frac{\mathbf{p}^2}{2M_{nc}} + B_0} + \text{regular}$$

Counting in S waves:
$a_0 \sim R_{\rm halo} \sim 1/\gamma_0$; $r_0 \sim R_{\rm core}$.
$r_0=0$ at LO.

One-slide p-wave review

$$\langle \mathbf{k} | t_1 | \mathbf{k}' \rangle = -\frac{6\pi}{m_R} \frac{\mathbf{k} \cdot \mathbf{k}'}{-\frac{1}{a_1} + \frac{1}{2} r_1 k^2 - i k^3}$$

Bethe (1949)

One-slide p-wave review

- For a short-ranged potential, if kR≲1:

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Wigner (1955); Hammer & Lee (2009); Nishida (2012)

- So low-energy resonance/bound state would seem to have to arise due to cancellation between $-1/a_1$ and $1/2\ r_1 k^2$ terms.

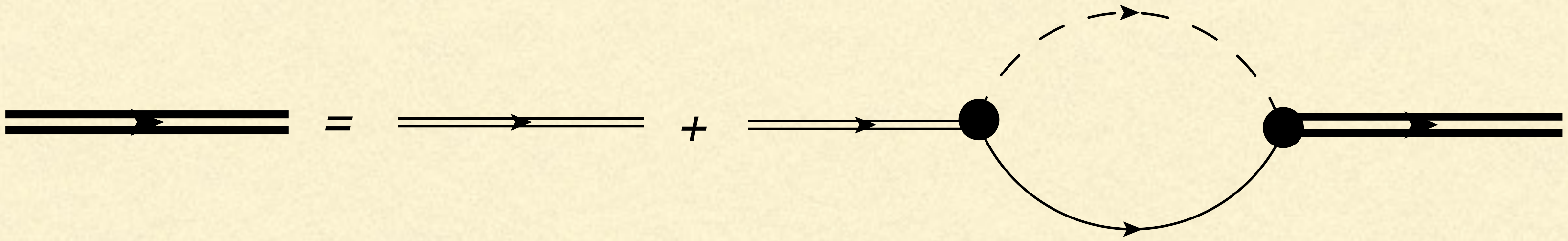
- $a_1 \sim R/M_{lo}^2$ gives $k_R \sim M_{lo}$

Bedaque, Hammer, van Kolck (2003)

Dressing the p-wave state

Bertulani, Hammer, van Kolck (2002); Bedaque, Hammer, van Kolck (2003)

- Proceed similarly for p-wave state as for s-wave state



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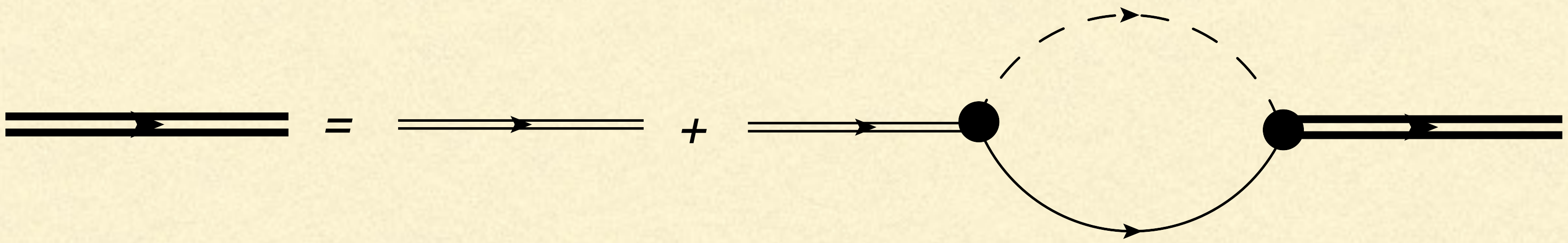

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- If a_l > 0 then pole is at $k=i\gamma_l$ with $B_l=\gamma_l^2/(2m_R)$:

$$D_\pi(p) = -\frac{3\pi}{m_R^2 g_1^2}\frac{2}{r_1 + 3\gamma_1}\frac{i}{p_0 - \mathbf{p}^2/(2M_{nc}) + B_1} + \text{regular}$$

A narrow p-wave resonance/bound state

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$$\langle \mathbf{k}|t_1|\mathbf{k}'\rangle = -\frac{12\pi}{m_R r_1}\frac{\mathbf{k}\cdot\mathbf{k}'}{k^2 - k_R^2} \qquad k_R^2 = \frac{2}{a_1 r_1}$$

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- Resonance width is $\sim E_R\, k_R/r_1$, so it is parametrically narrow. Need to resum width if $k^2-k_R^2$ gets small

P-wave FSI in $\gamma_{E1} + {}^{11}\text{Be} \rightarrow {}^{10}\text{Be} + n$

Typel & Baur, Phys. Rev. Lett. 93, 142502 (2004); Nucl. Phys. A759, 247 (2005); Eur. Phys. J. A 38, 355 (2008)

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$$k^3 \cot \delta_1 = -1/2\, r_1 (k^2 + \gamma_1{}^2) \Rightarrow \delta_1 \sim R_{core}/R_{halo} \text{ if } k \sim 1/R_{halo} \sim \gamma_1.$$

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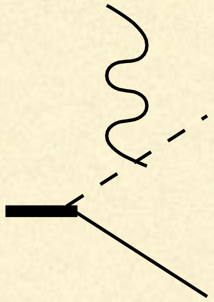
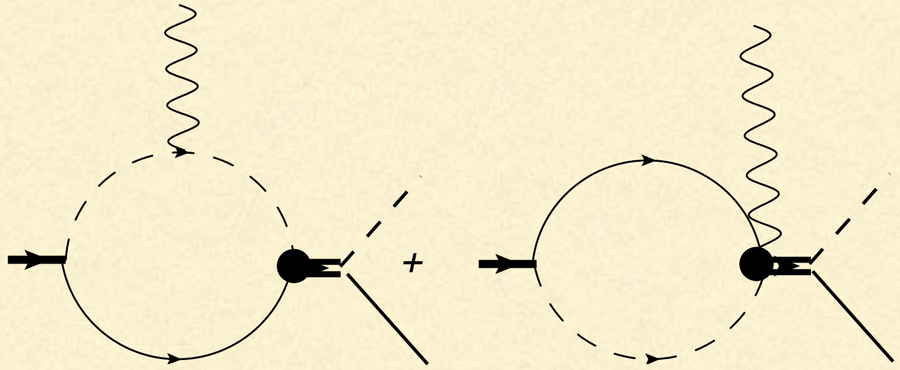


LO



NLO

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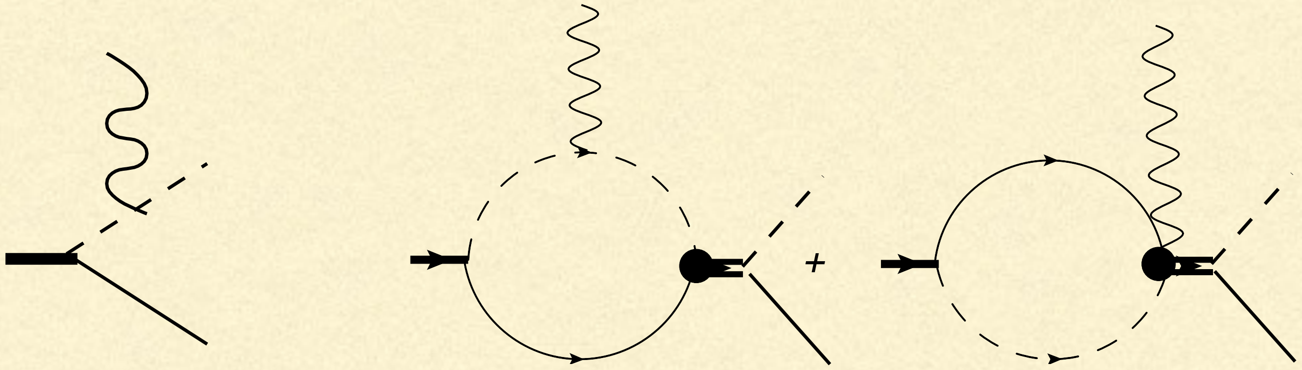
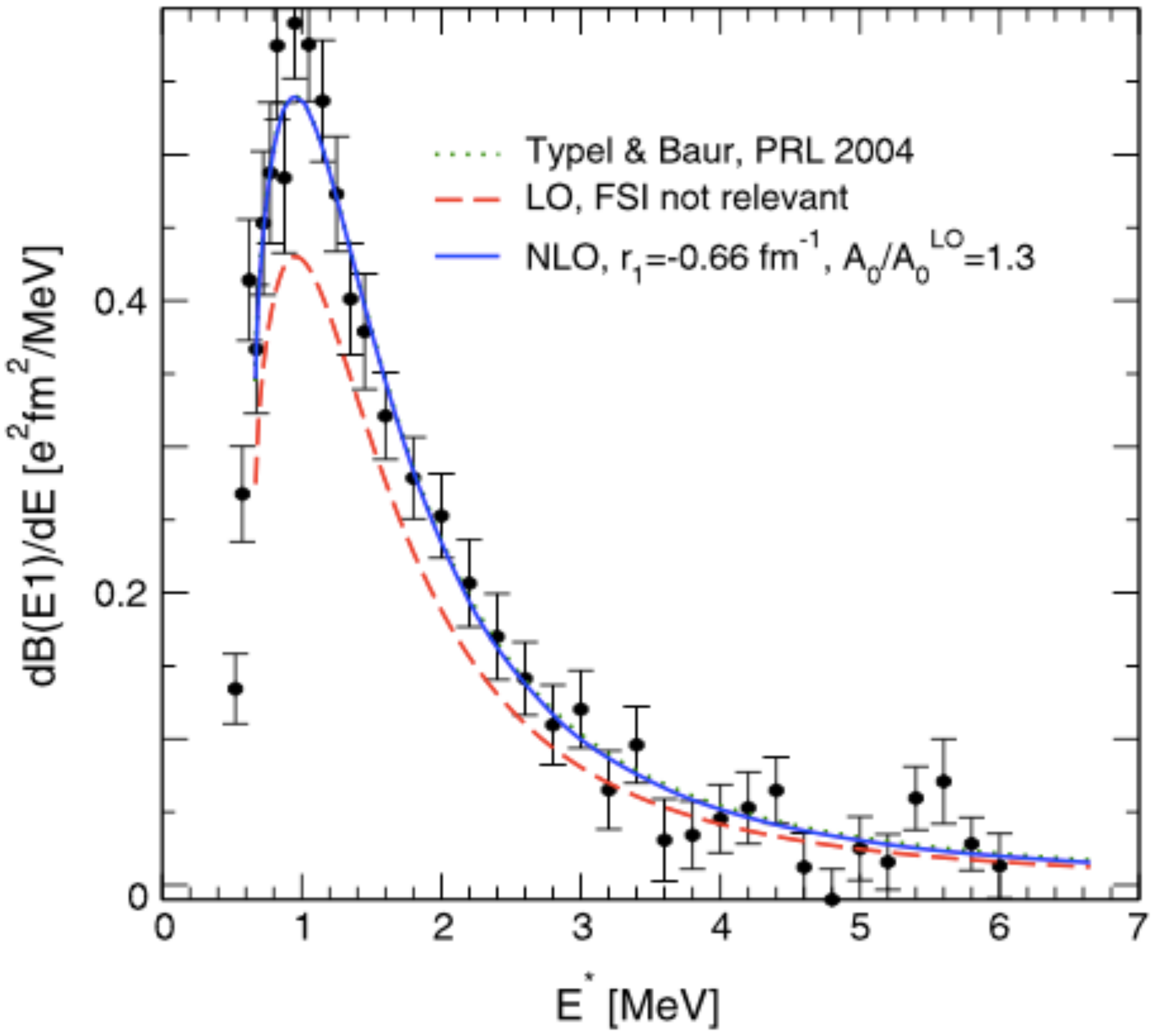


LO NLO

- Need both γ_1 and $r_1 \equiv A_1$ at NLO in this observable. A_0 also becomes a free parameter at NLO: fit it to Coulomb dissociation data

Coulomb dissociation of ^{11}Be: result



Data: Palit et al., 2003
Analysis: Hammer, Phillips. NPA, 2011

- Reasonable convergence
- Information on value of r_0 through fitting of A_0:

r_0=2.7 fm

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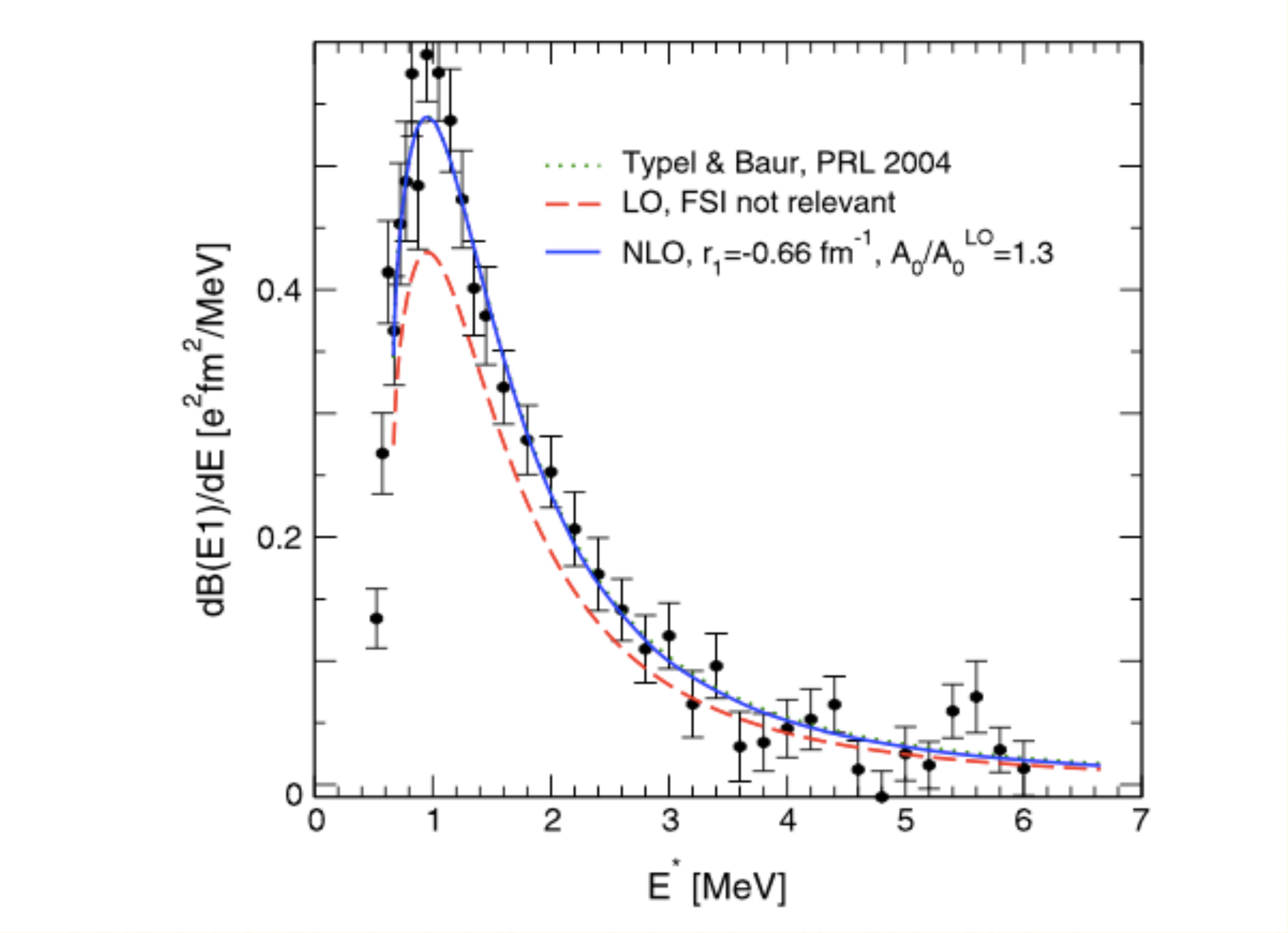

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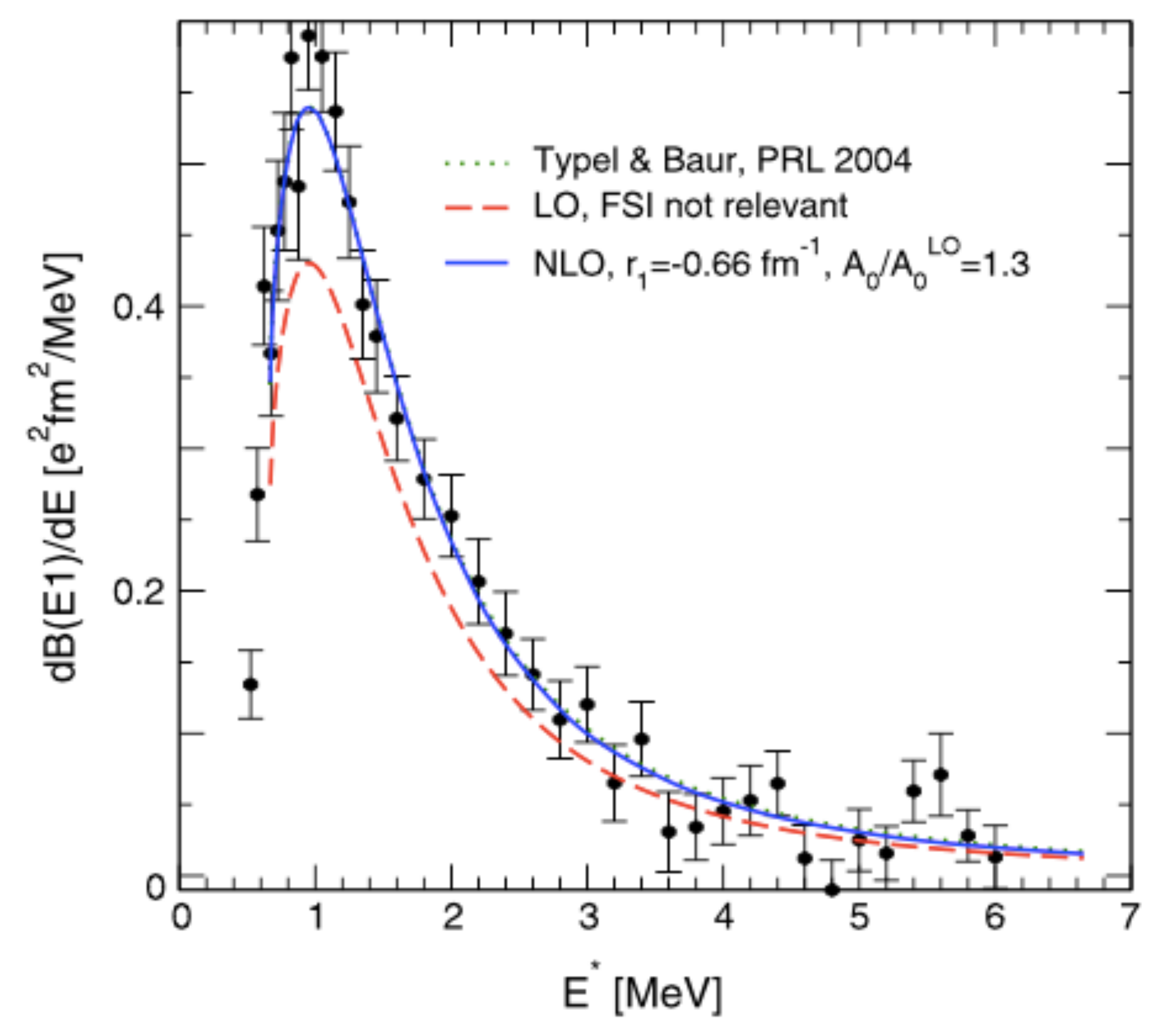

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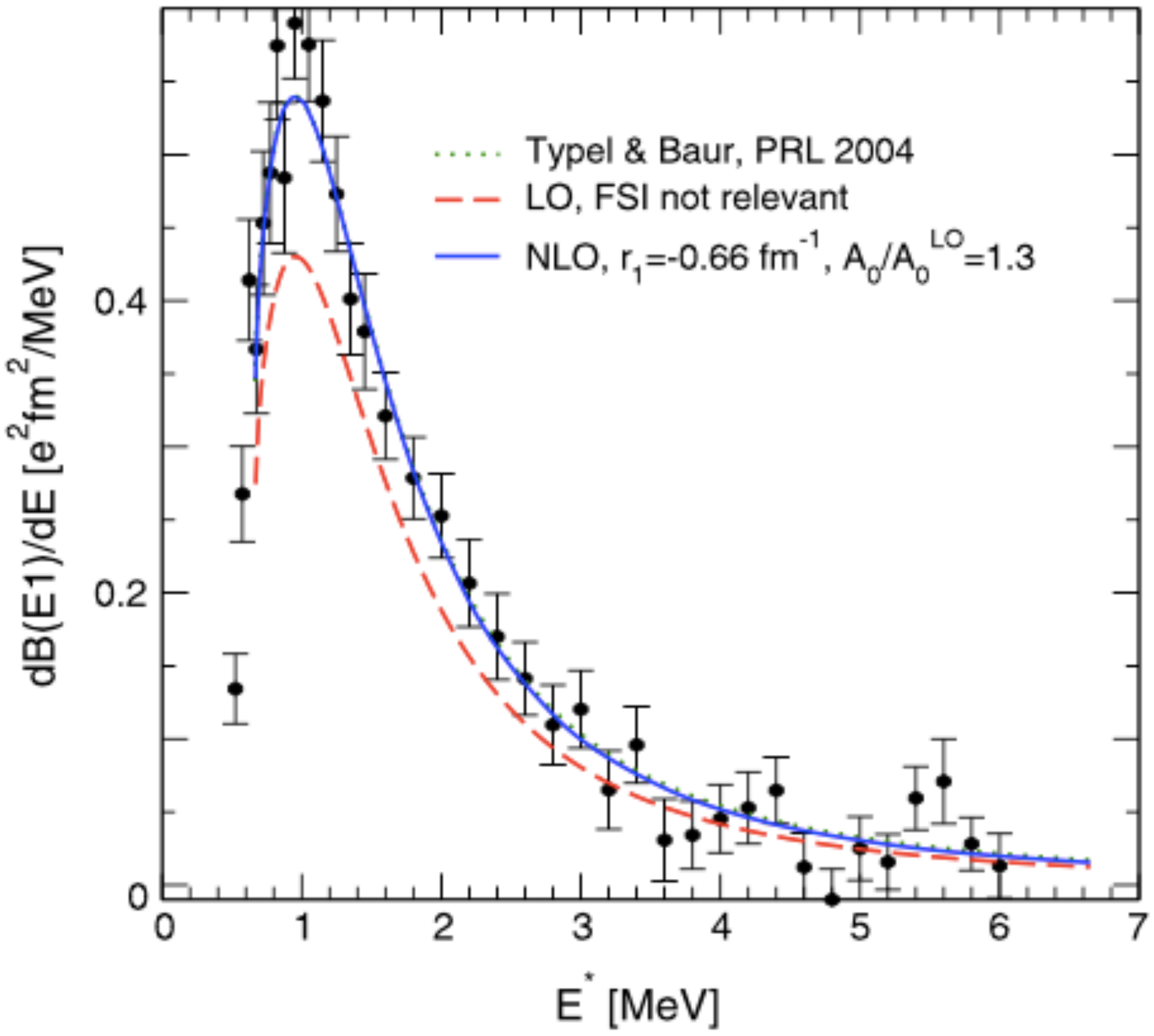

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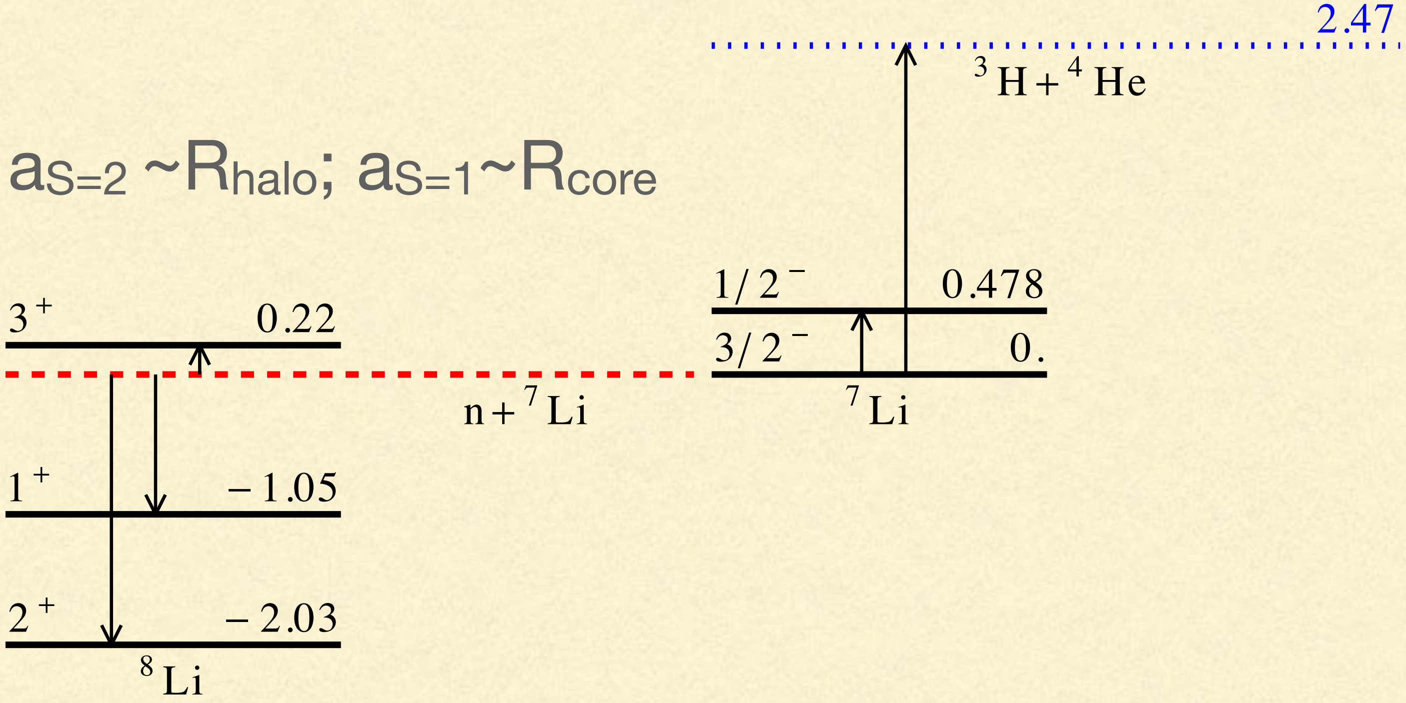
Use of ab initio input, e.g. for ANC?

^{7}Li + n→^{8}Li + γ_{E1}

- ^{7}Li ground state is $3/2^-$: S-wave n scattering in $^5S_2$ and $^3S_1$

$a_{S=2} \sim R_{halo}$; $a_{S=1} \sim R_{core}$



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- $a_{S=2}$=-3.63(5) fm, $a_{S=1}$=0.87(7) fm

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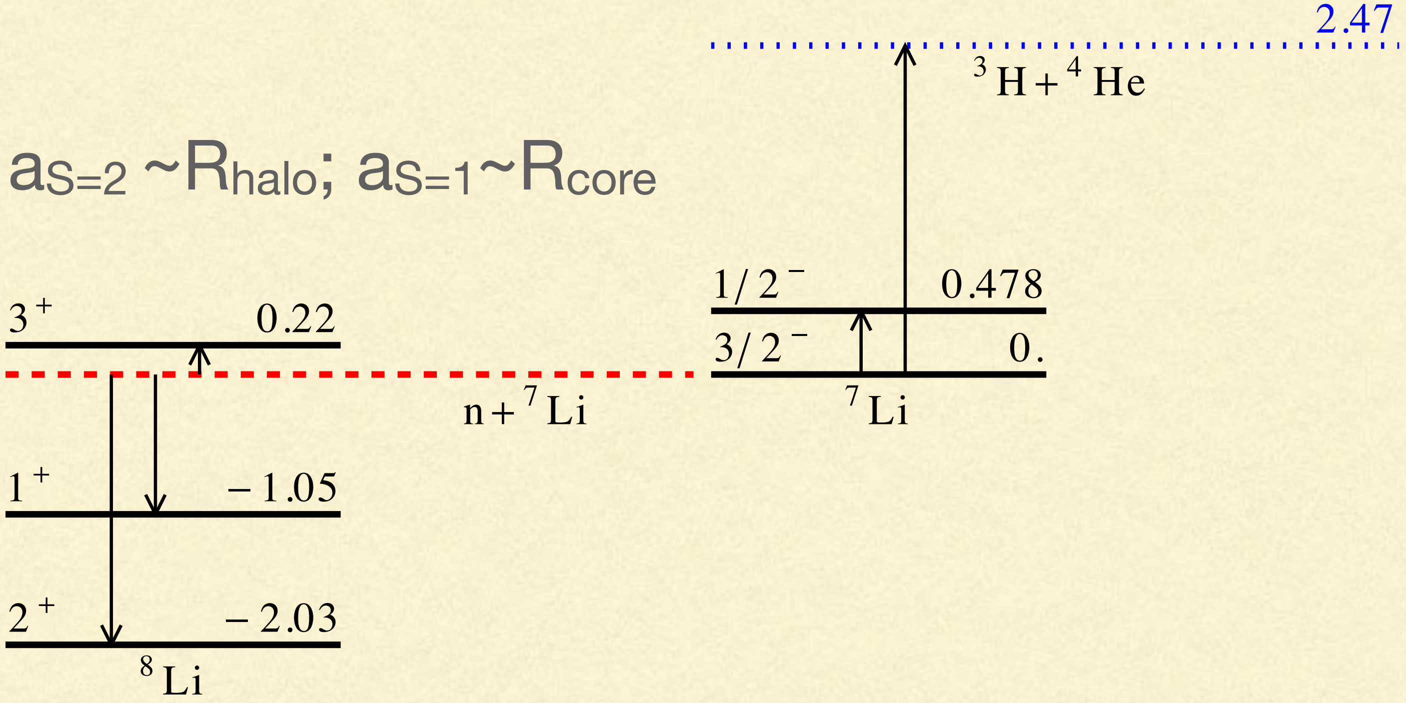
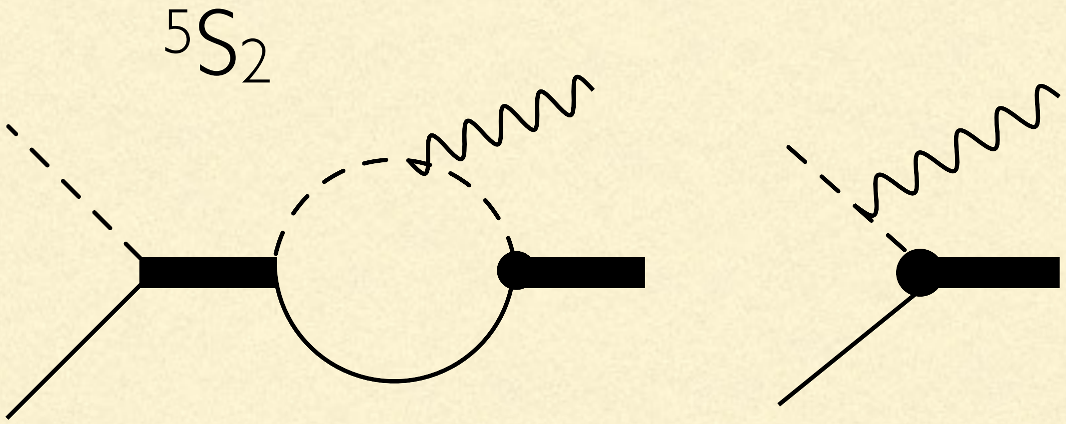
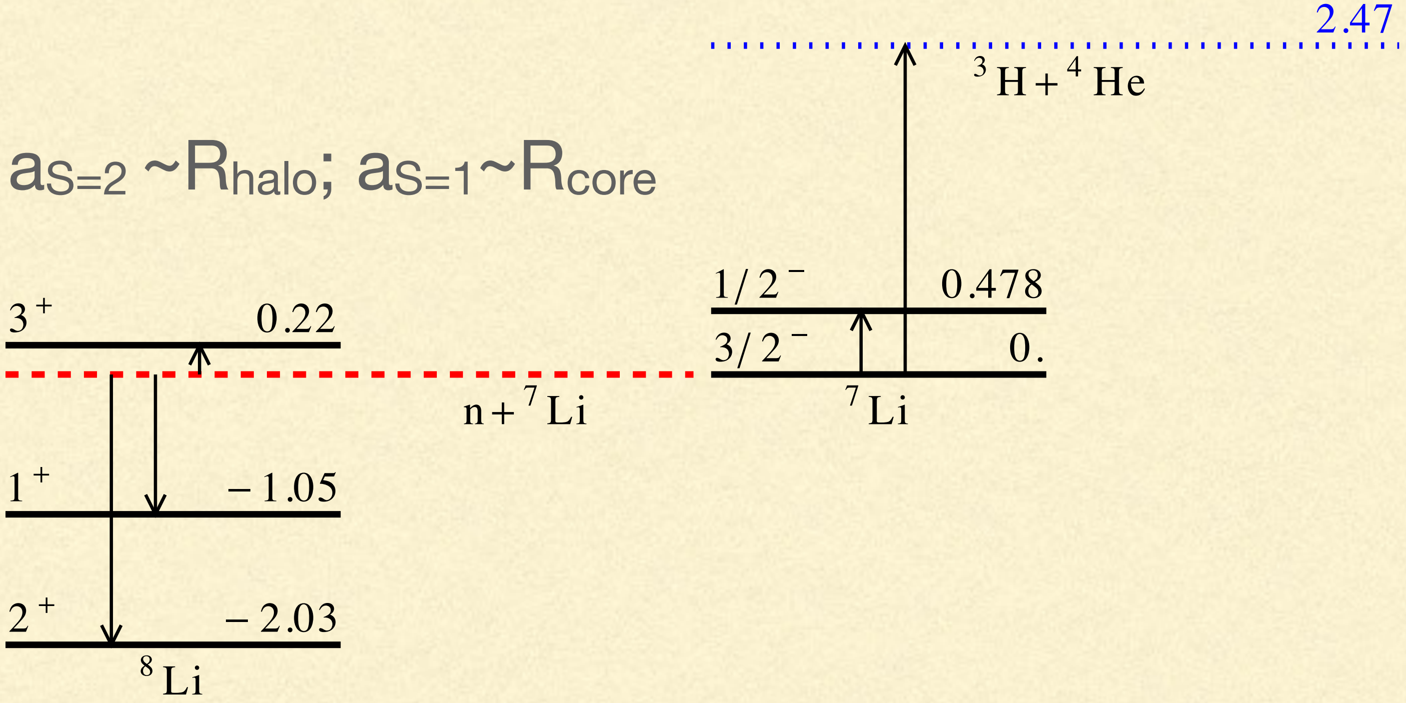


^{7}Li + n→^{8}Li + γ_{E1}

- ^{7}Li ground state is 3/2$^-$: S-wave n scattering in $^5S_2$ and $^3S_1$
- $a_{S=2}$=-3.63(5) fm, $a_{S=1}$=0.87(7) fm

$a_{S=2}$ ~R_{halo}; $a_{S=1}$~R_{core}





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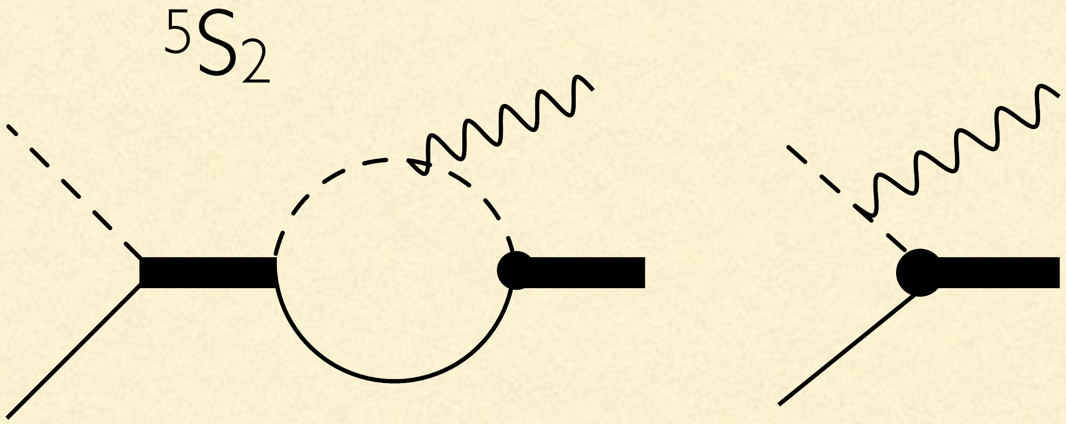

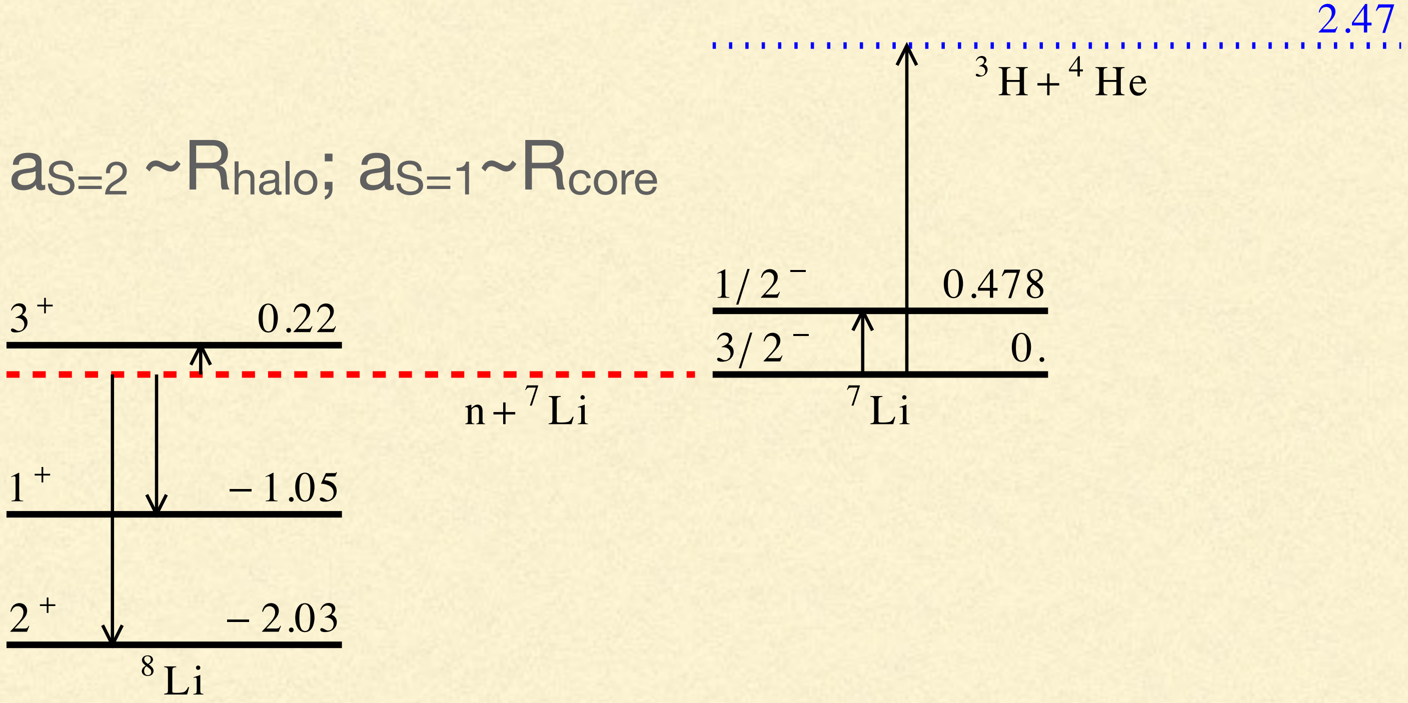

- LO calculation: S=2 (with ISI) and S=1 into P-wave bound state

$$\mathrm{E1} \propto \int_0^\infty dr\, u_0(r) r u_1(r);$$

$$u_0(r) = 1 - \frac{r}{a}; u_1(r) = A_1 \exp(-\gamma_1 r)\left(1 + \frac{1}{\gamma_1 r}\right)$$

Fixing ^{8}Li parameters

- ^{8}Li ground state is 2^+: both $^5P_2$ and $^3P_2$ components
- ^{8}Li first excited state: 1^+, bound by 1.05 MeV

Zhang, Nollett, Phillips, PRC (2014)

c.f. Rupak, Higa, PRL 106, 222501 (2011);
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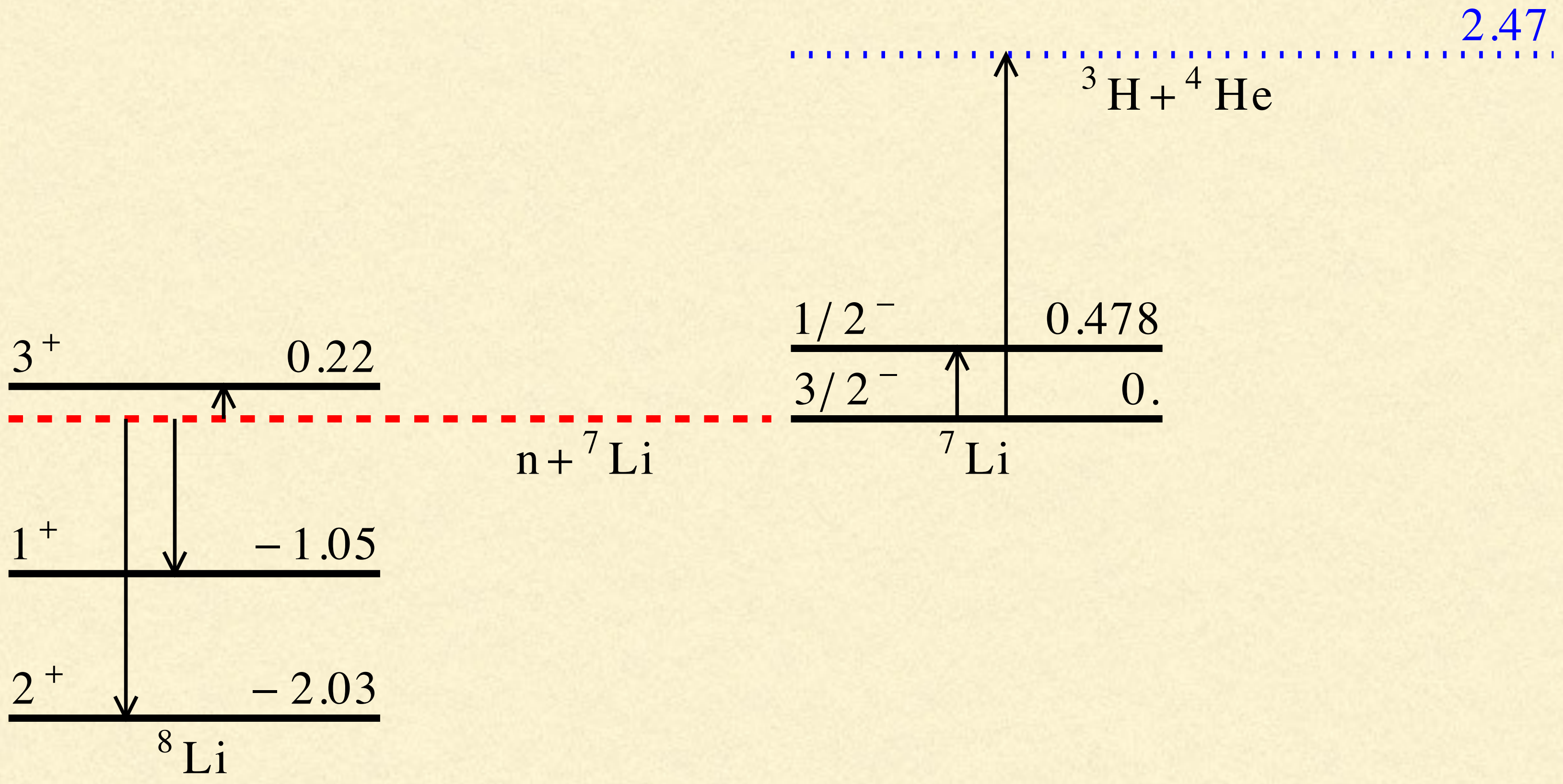

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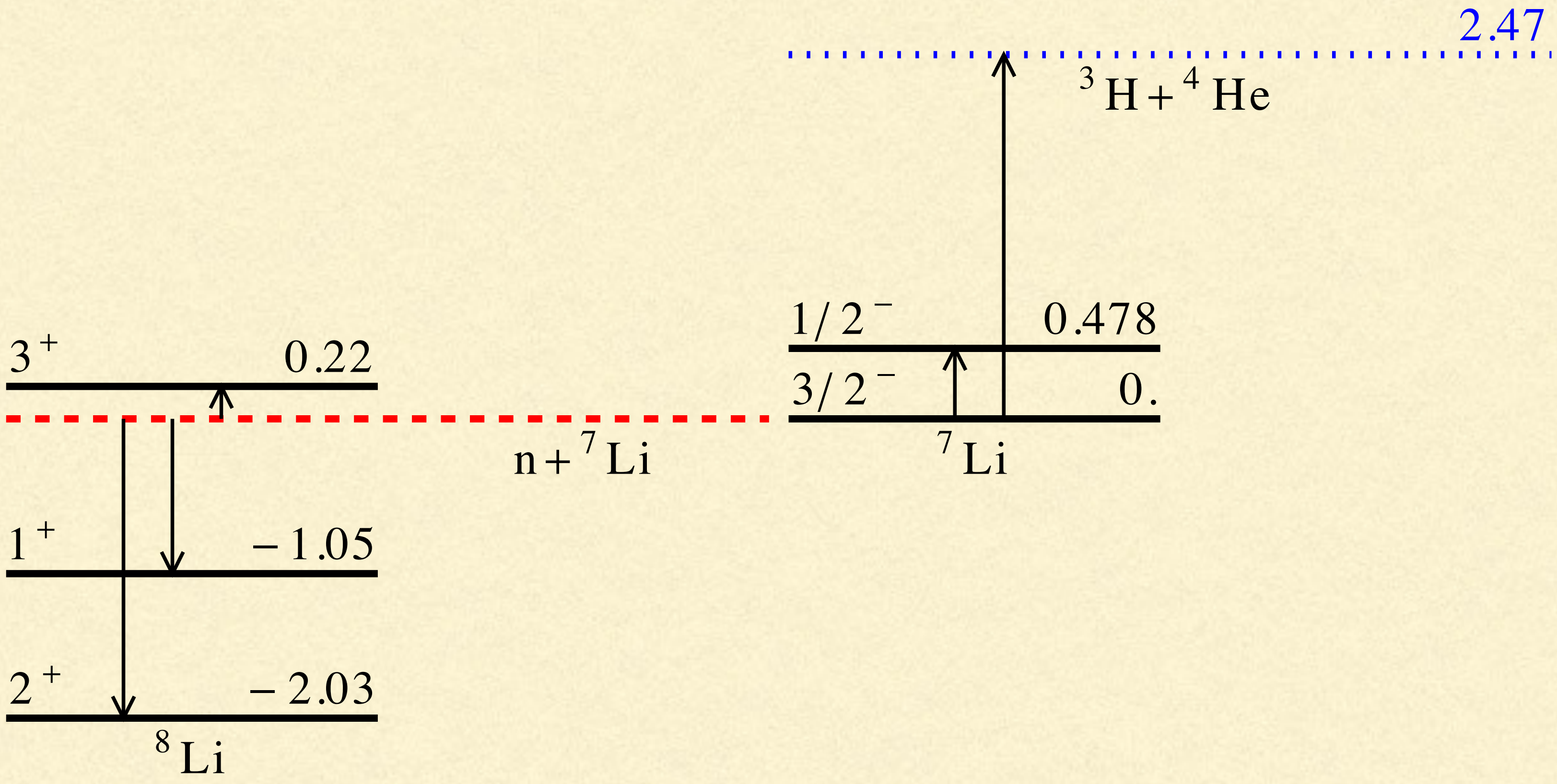
- Input at LO: B_1=2.03 MeV; B_1^*=1.05 MeV⇒γ_1=58 MeV; γ_1^*=42 MeV.

$\gamma_1 \sim 1/R_{halo}$



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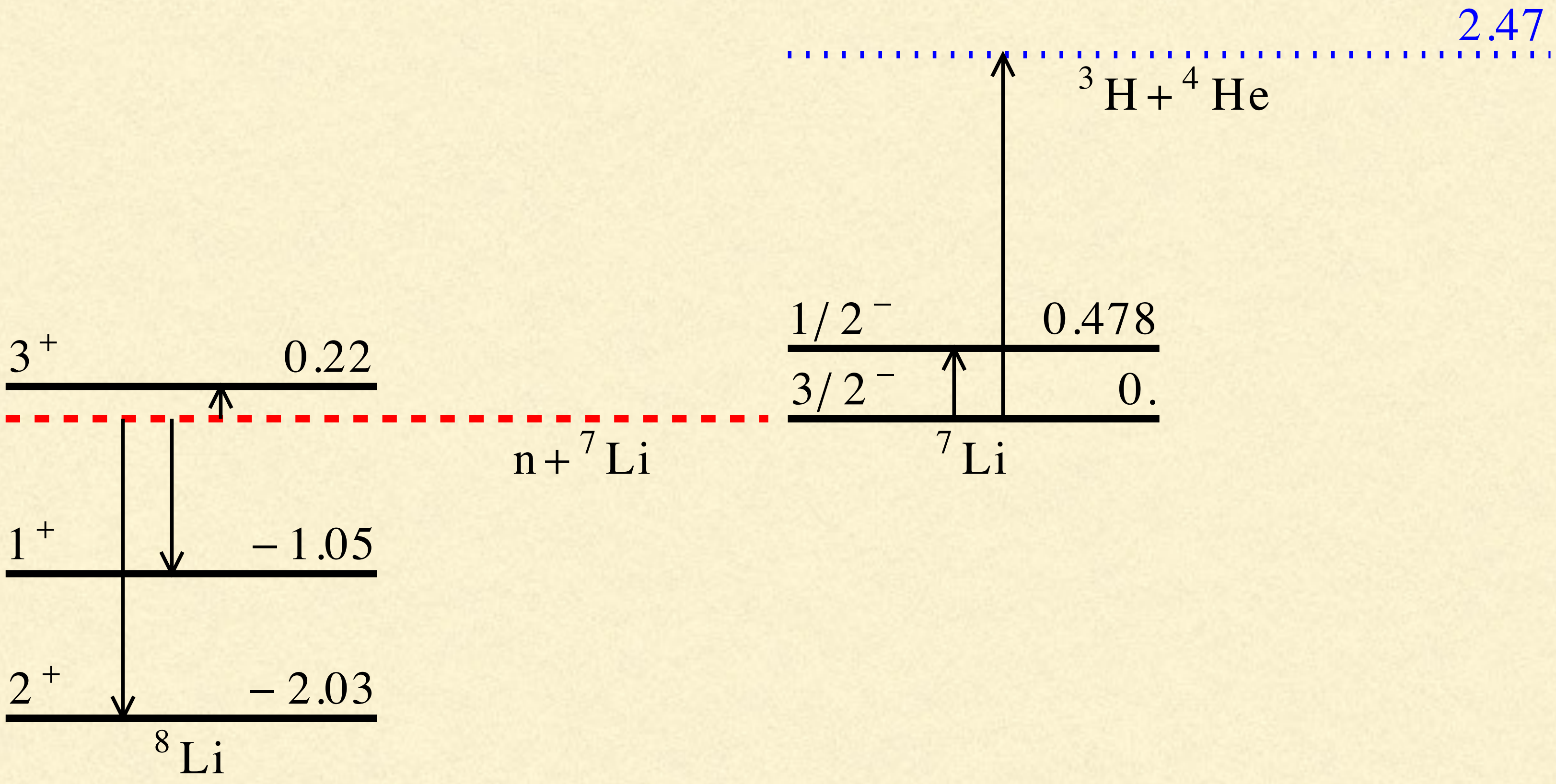

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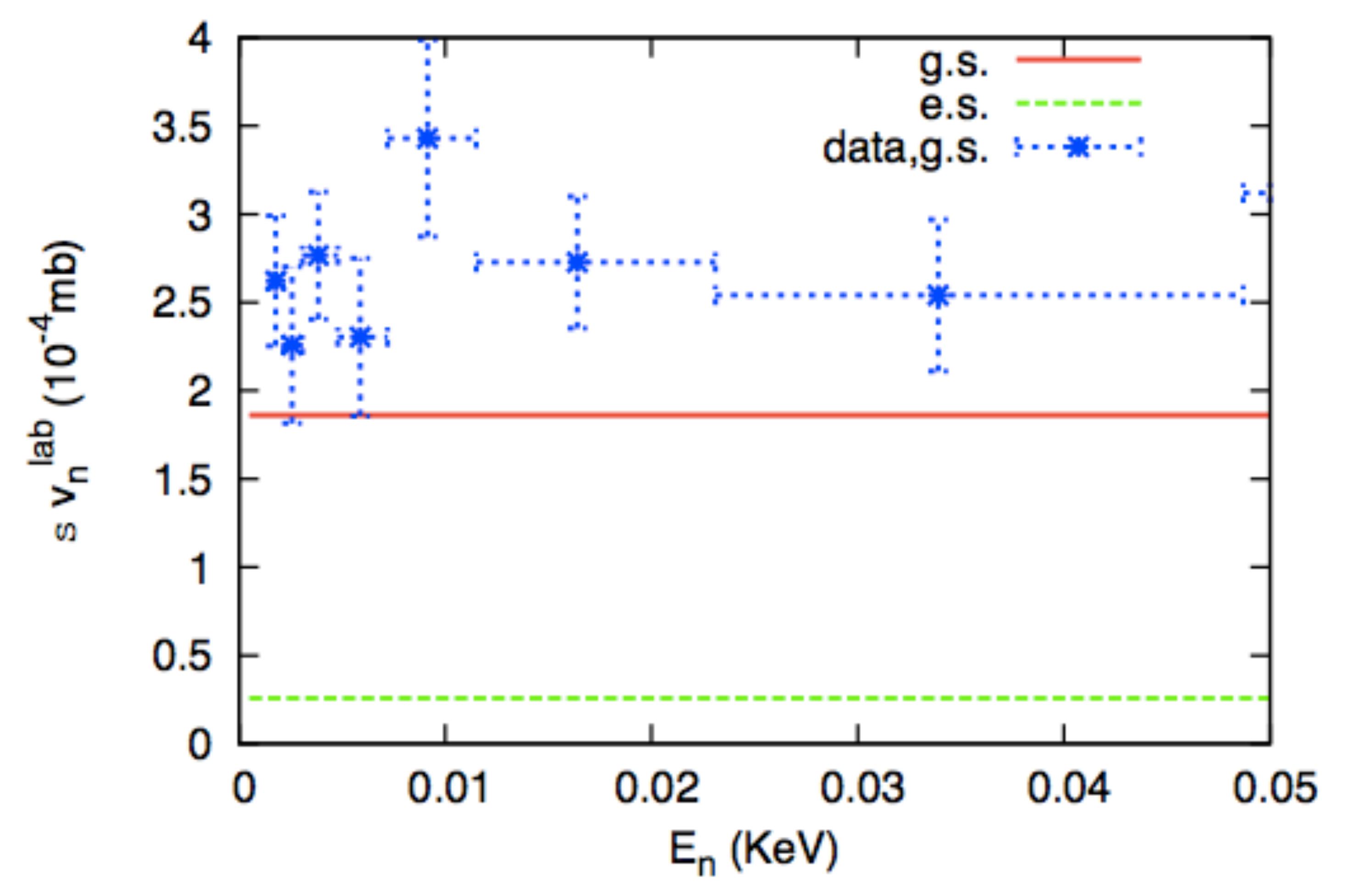
	$A_{(3P2)}$	$A_{(5P2)}$	$A_{(3P2^*)}$	$A_{(3P1)}^*$	$A_{(5P1)}^*$
Nollett	-0.283(12)	-0.591(12)	-0.384(6)	0.220(6)	0.197(5)
Trache	-0.284(23)	-0.593(23)		0.187(16)	0.217(13)

LO results for ^{7}Li + n$\rightarrow$^{8}Li + γ_{E1}



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Data: Barker (1996), cf. Nagai et al. (2005)

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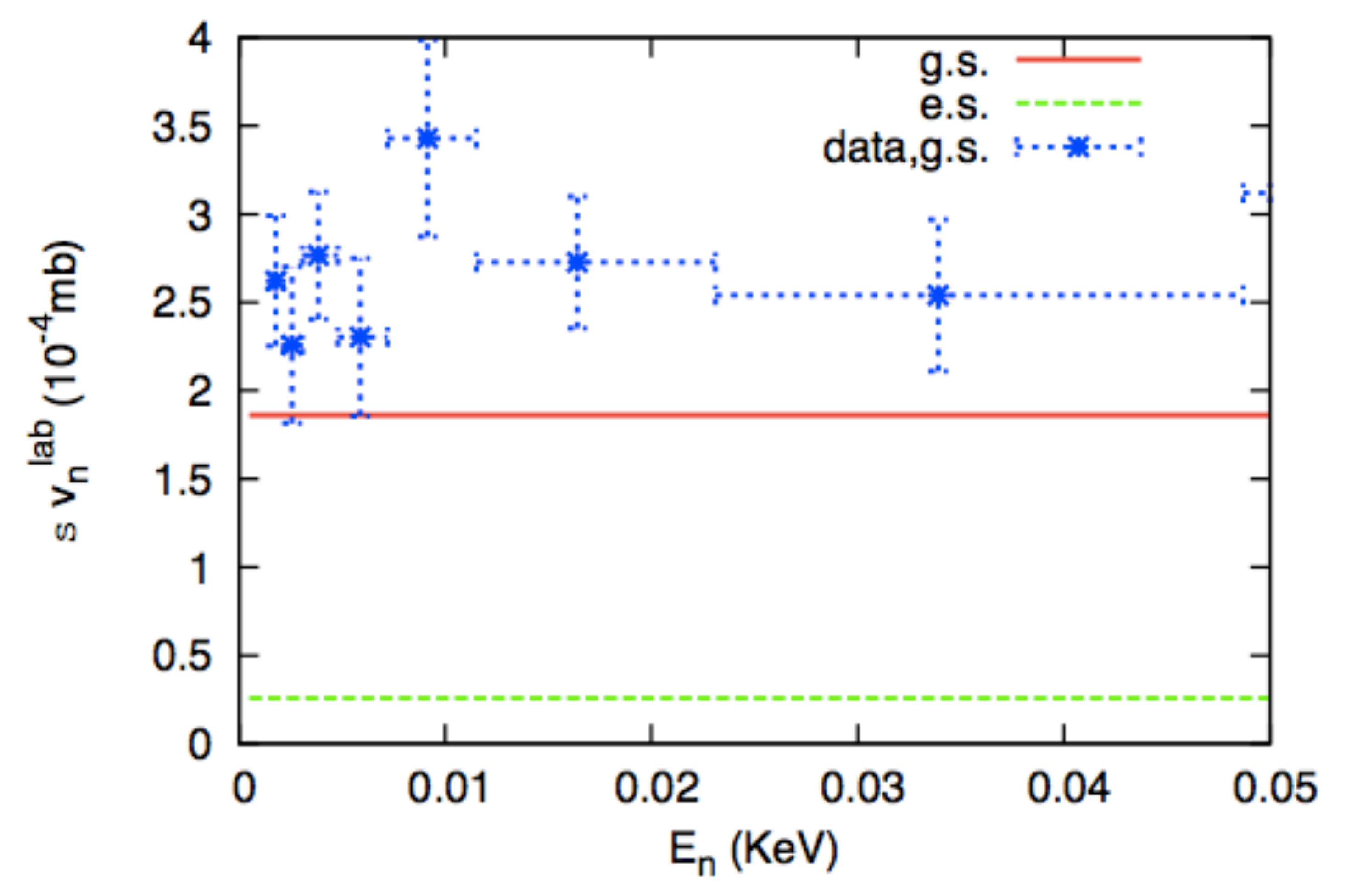

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Experiment > 0.86

Barker, 1996

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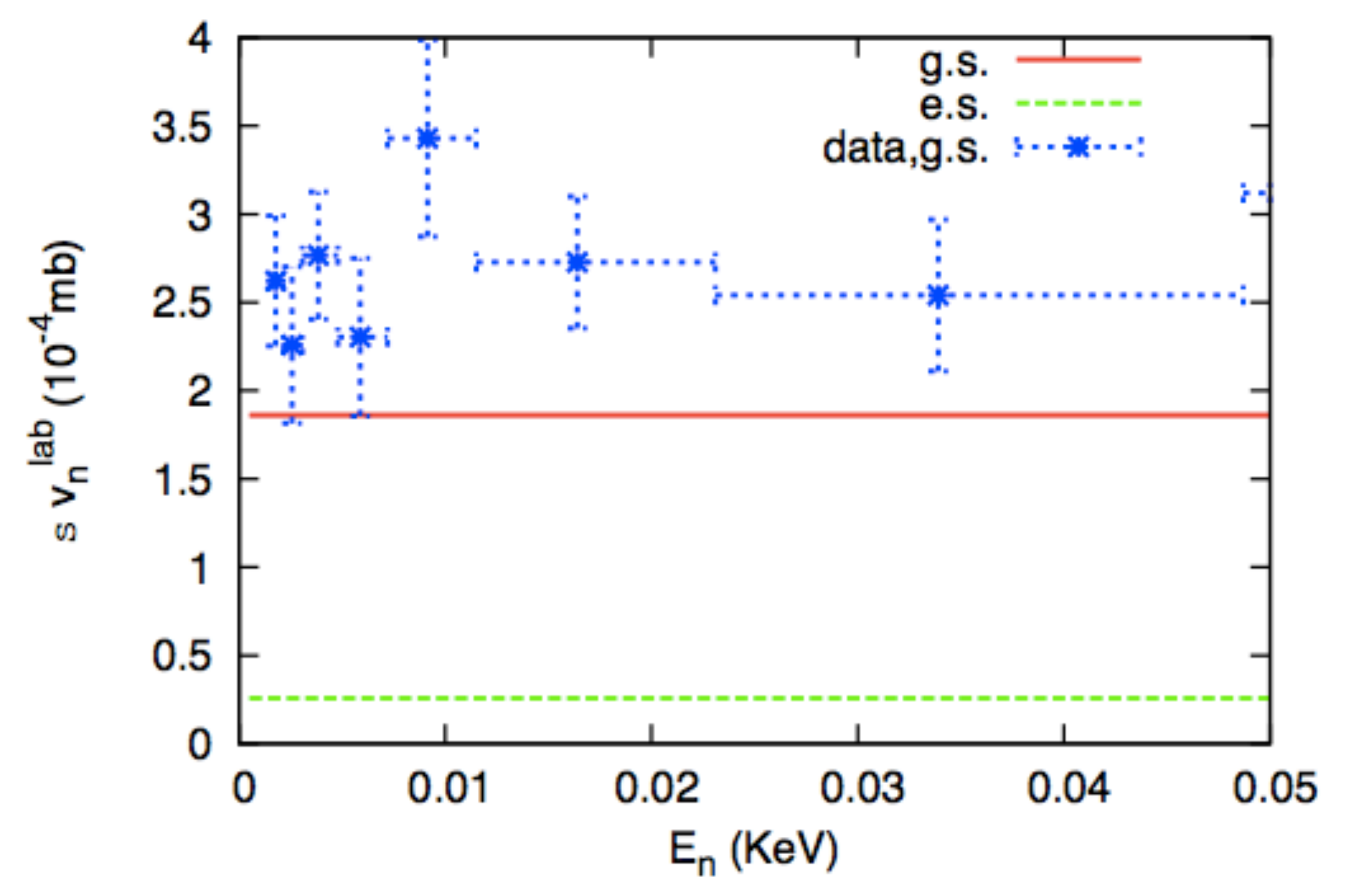

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Lynn et al., 1991

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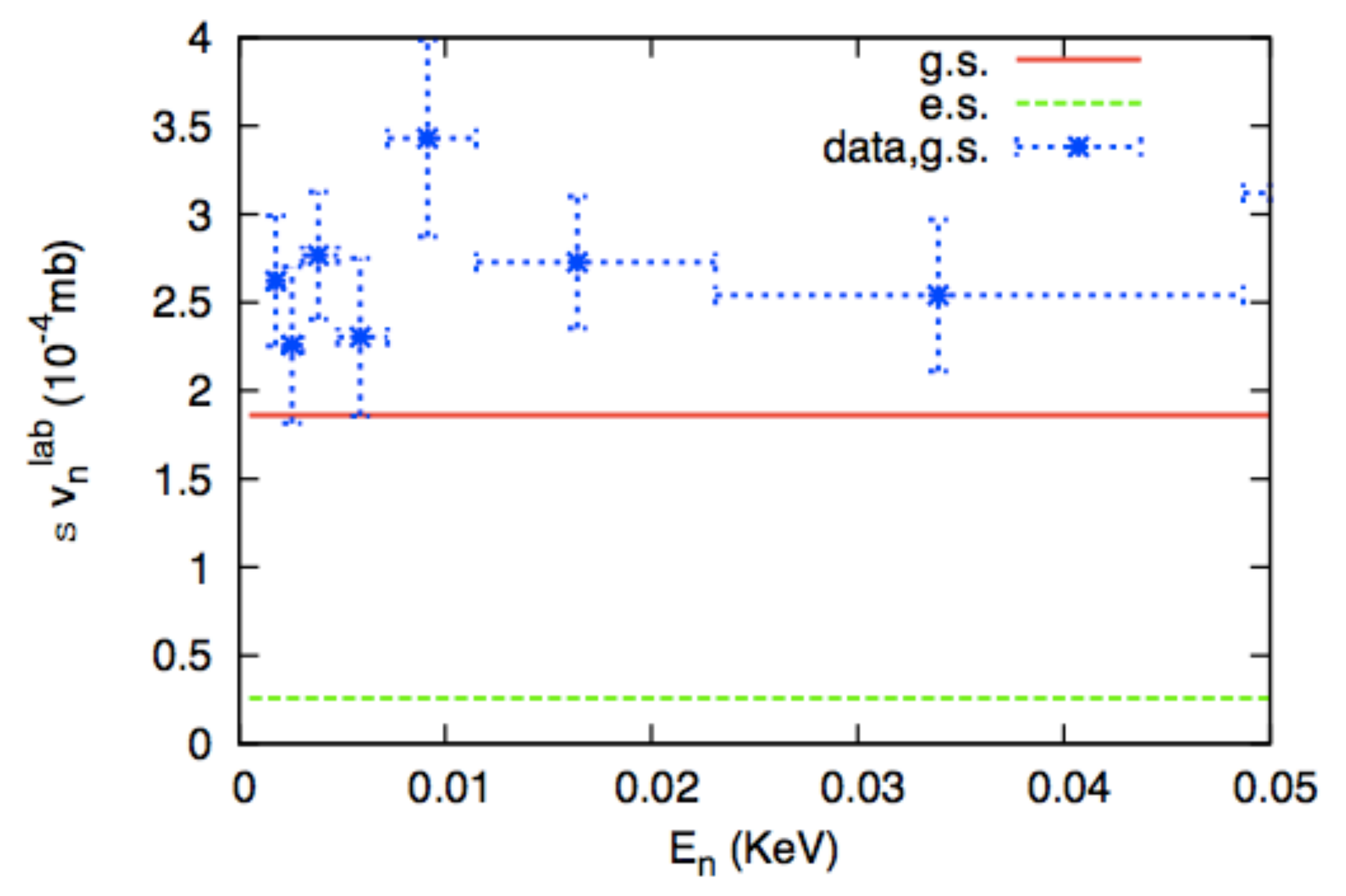

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Dynamics **predicted** through *ab initio* input

Data situation

42 data points for 100 keV $< E_{c.m.} <$ 500 keV	CMEs
Junghans (BE1 and BE3)	2.7% and 2.3%
Fillipone	11.25%
Baby	5%
Hammache (1998 and 2001)	2.2% (1998)

- Subtract M1 resonance: negligible impact at 500 keV and below
- Deal with CMEs by introducing five additional parameters, ξ_j

Building the pdf

- Bayes:

$$\mathrm{pr}\left(\vec{g},\{\xi_i\}|D;T;I\right)=\mathrm{pr}\left(D|\vec{g},\{\xi_i\};T;I\right)\mathrm{pr}\left(\vec{g},\{\xi_i\}|I\right),$$

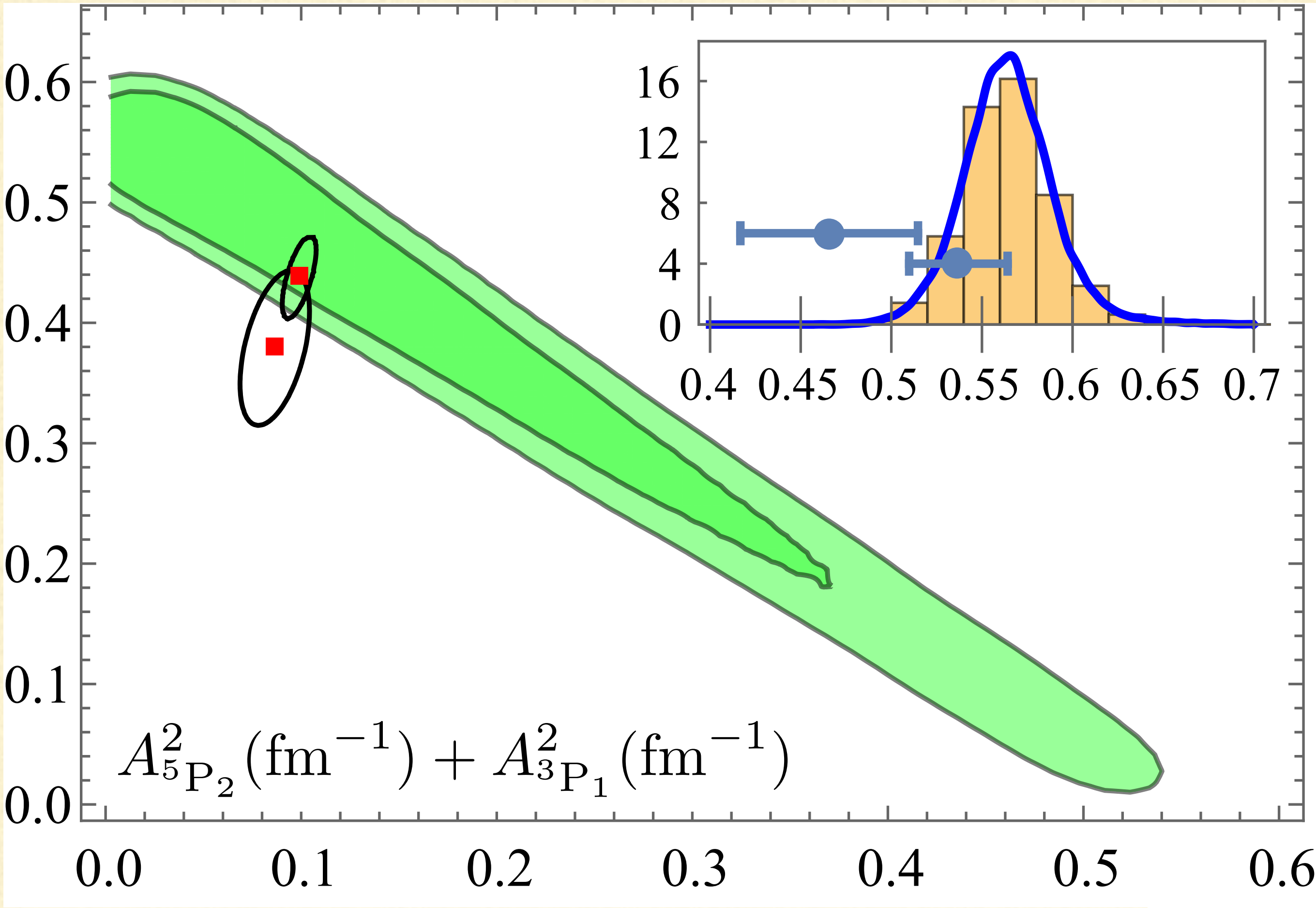
- First factor: likelihood

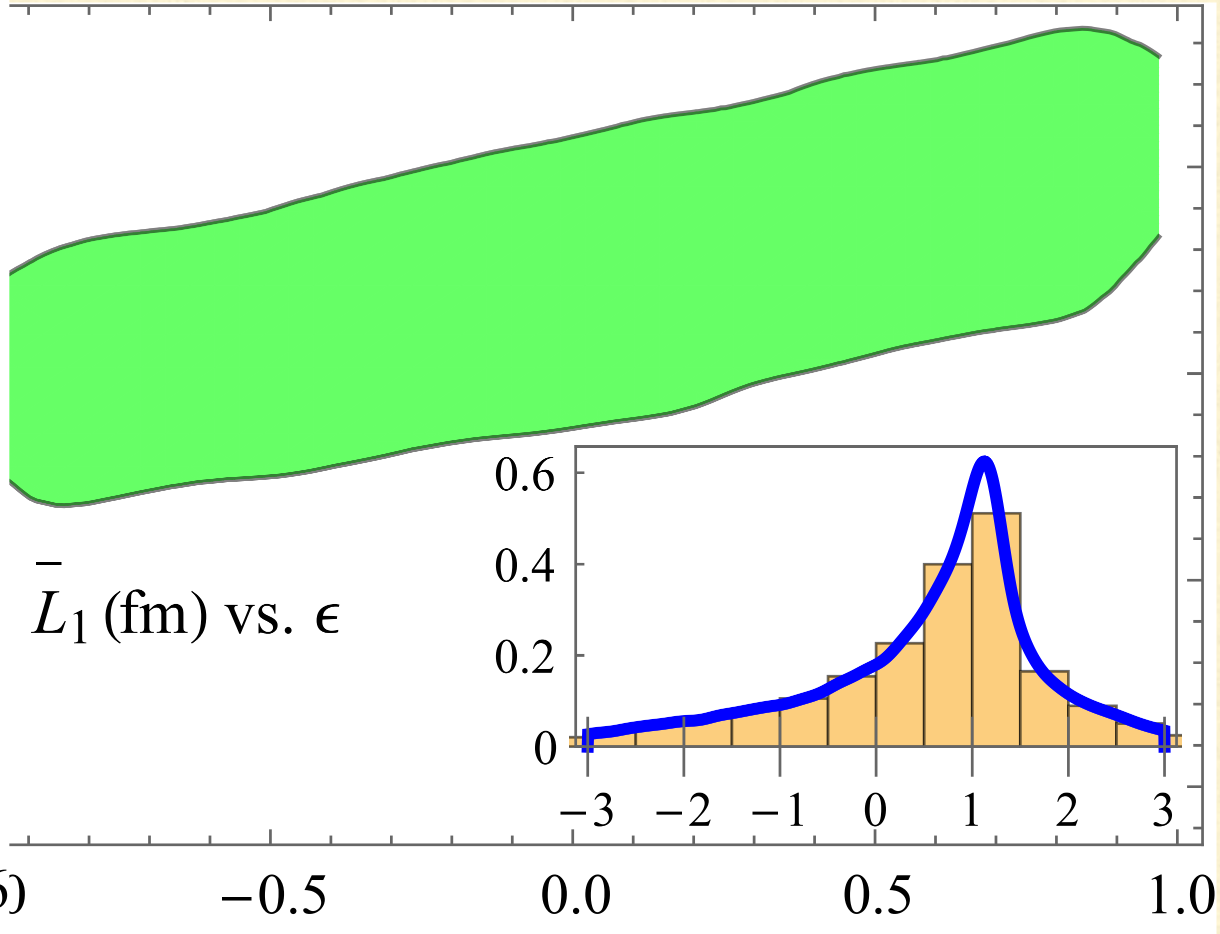
$$\ln\mathrm{pr}\left(D|\vec{g},\{\xi_i\};T;I\right)=c-\sum_{j=1}^{N}\frac{\left[(1-\xi_j)S(\vec{g};E_j)-D_j\right]^2}{2\sigma_j^2},$$

- Second factor: priors
  - Independent gaussian priors for ξ_j, centered at zero and with width=CME
  - Gaussian priors for $a_{S=1}$ and $a_{S=2}$, based on Angulo et al. measurement
  - All other EFT parameters assigned flat priors, corresponding to natural ranges
- No s-wave resonance below 600 keV

Marginalizing→pdfs

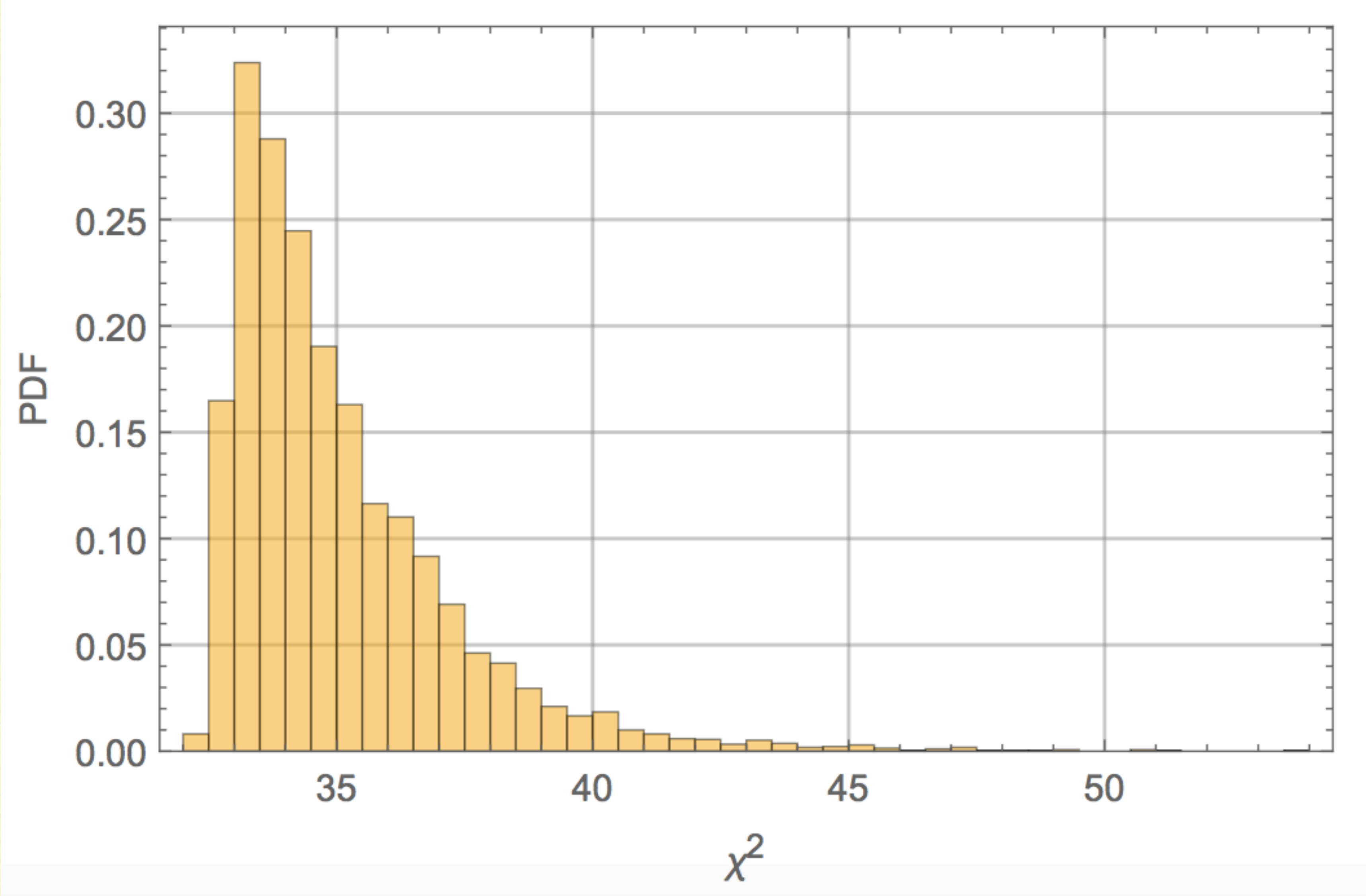
$$\mathrm{pr}\left(g_1, g_2|D;T;I\right) = \int \mathrm{pr}\left(\vec{g}, \{\xi_i\}|D;T;I\right)\, d\xi_1 \ldots d\xi_5 dg_3 \ldots dg_9$$





- ANCs are highly correlated but sum of squares strongly constrained
- One spin-1 short-distance parameter: $0.33\ \bar{L}_1/(\mathrm{fm}^{-1}) - \epsilon_1$

More questions we can answer



42 data points,
7 parameters "fit"
to these data,
5 $\xi_{i,}$'s fixed to their
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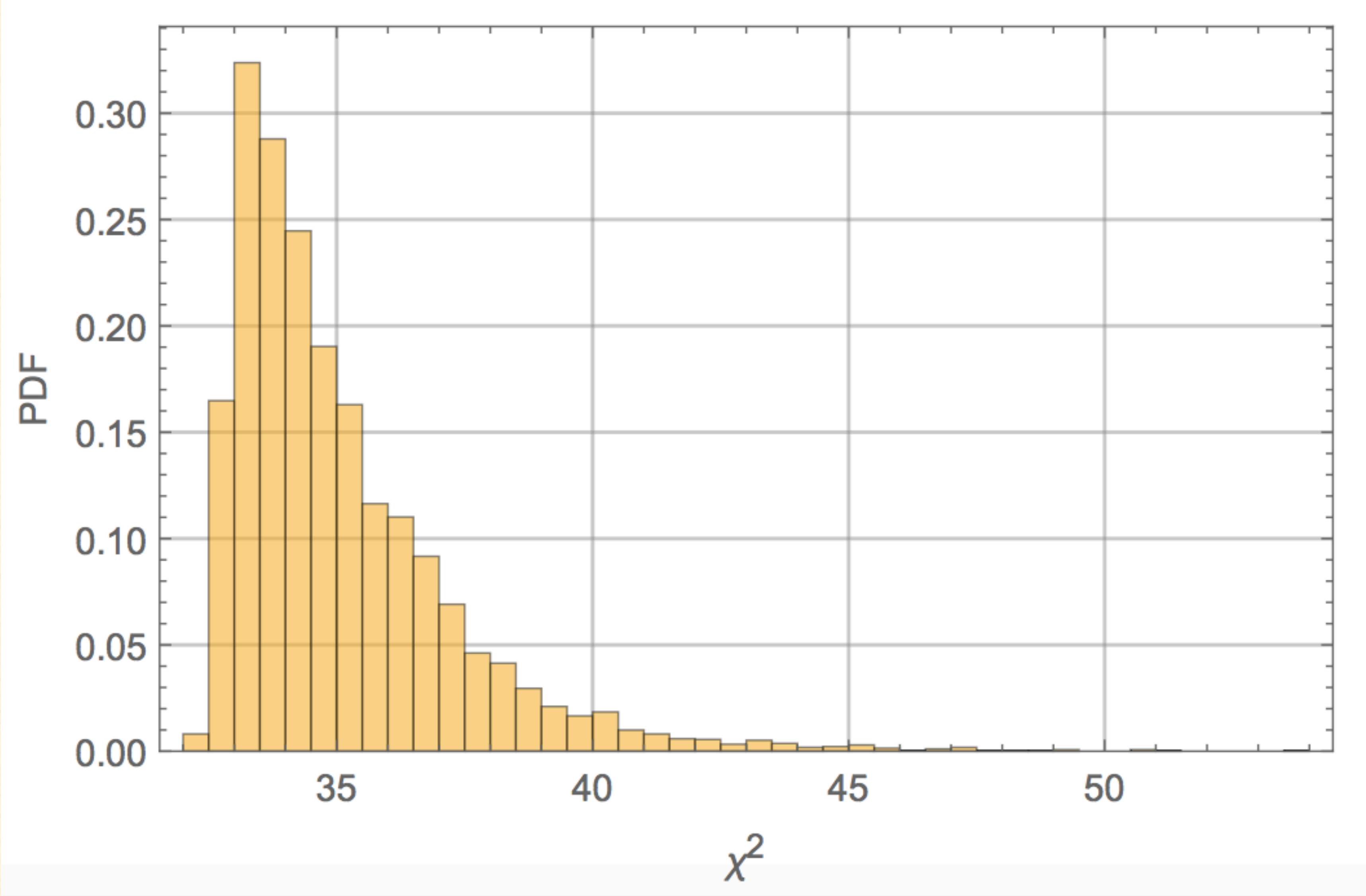
- Is it a “good fit”?



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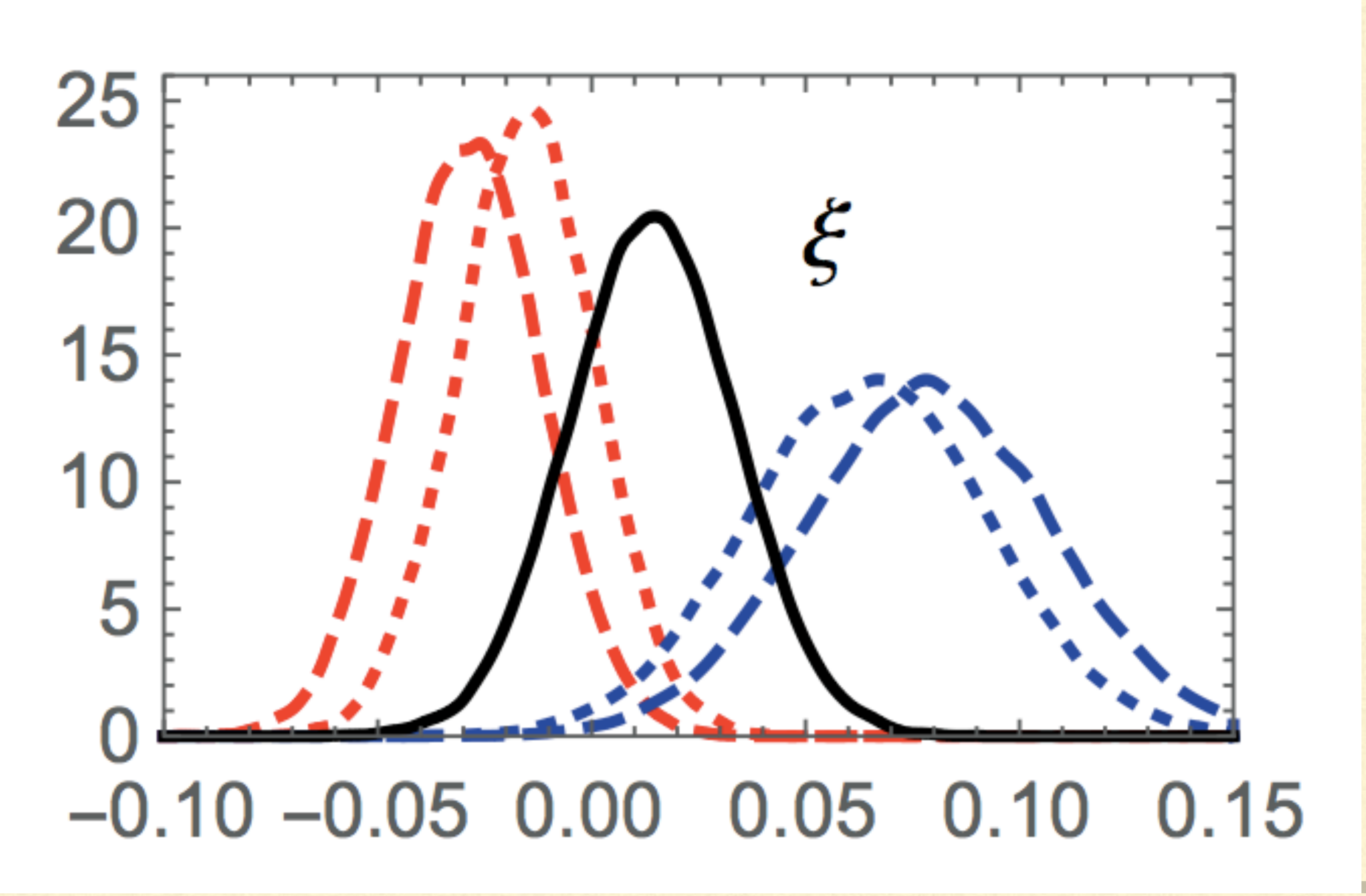
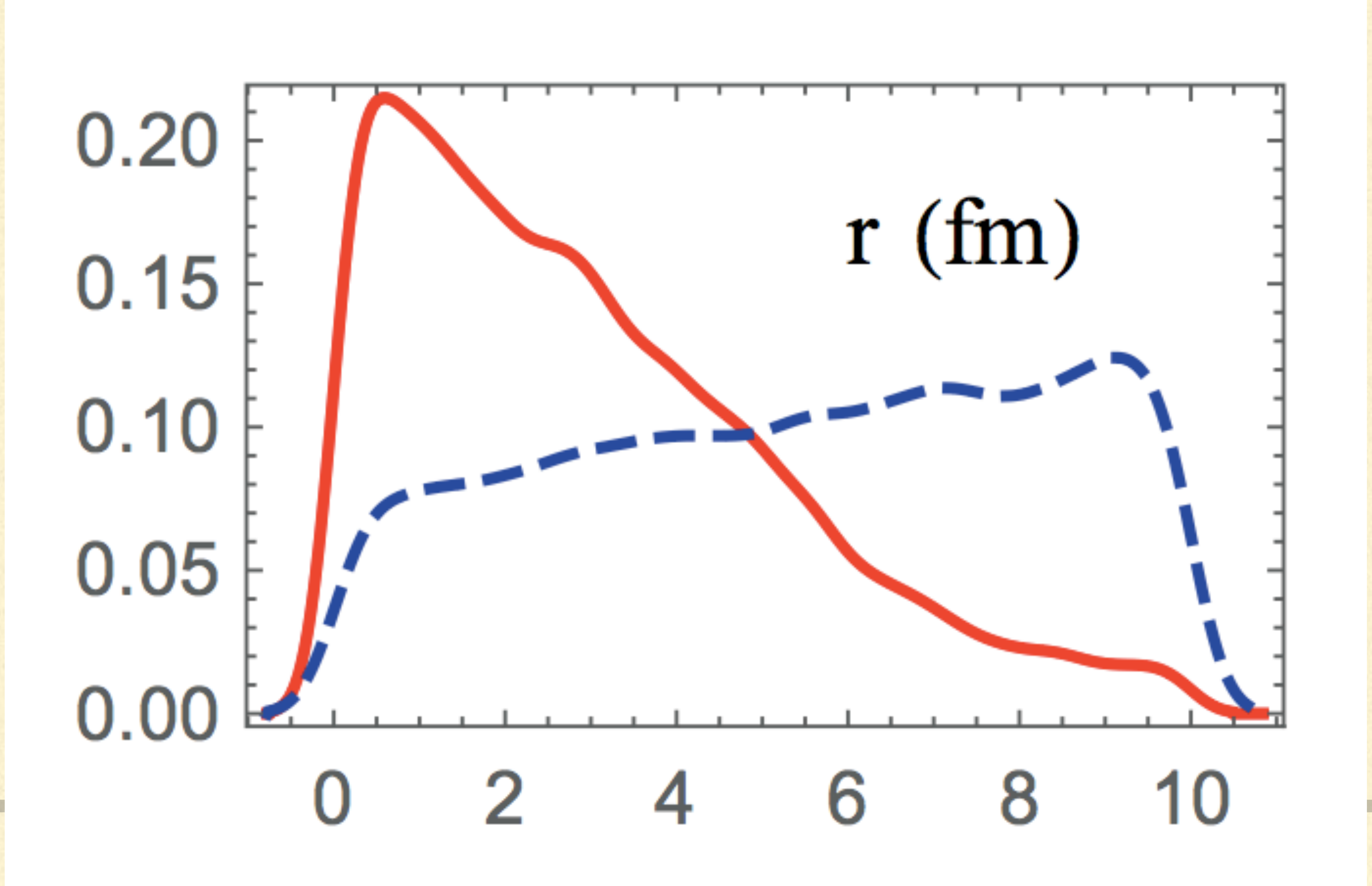


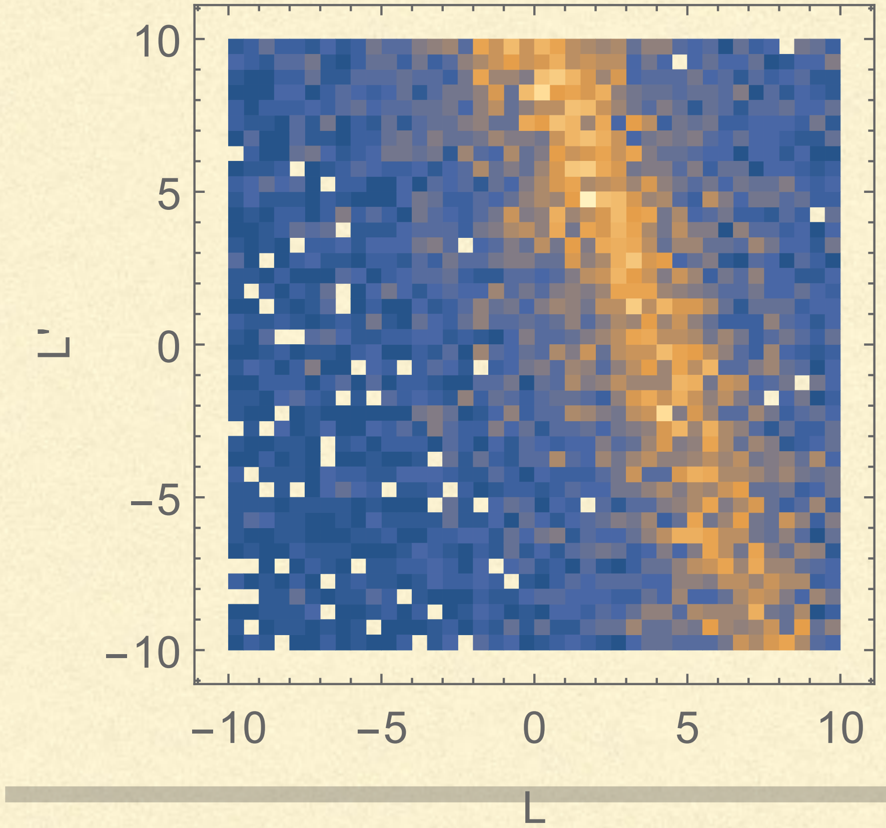
More questions we can answer

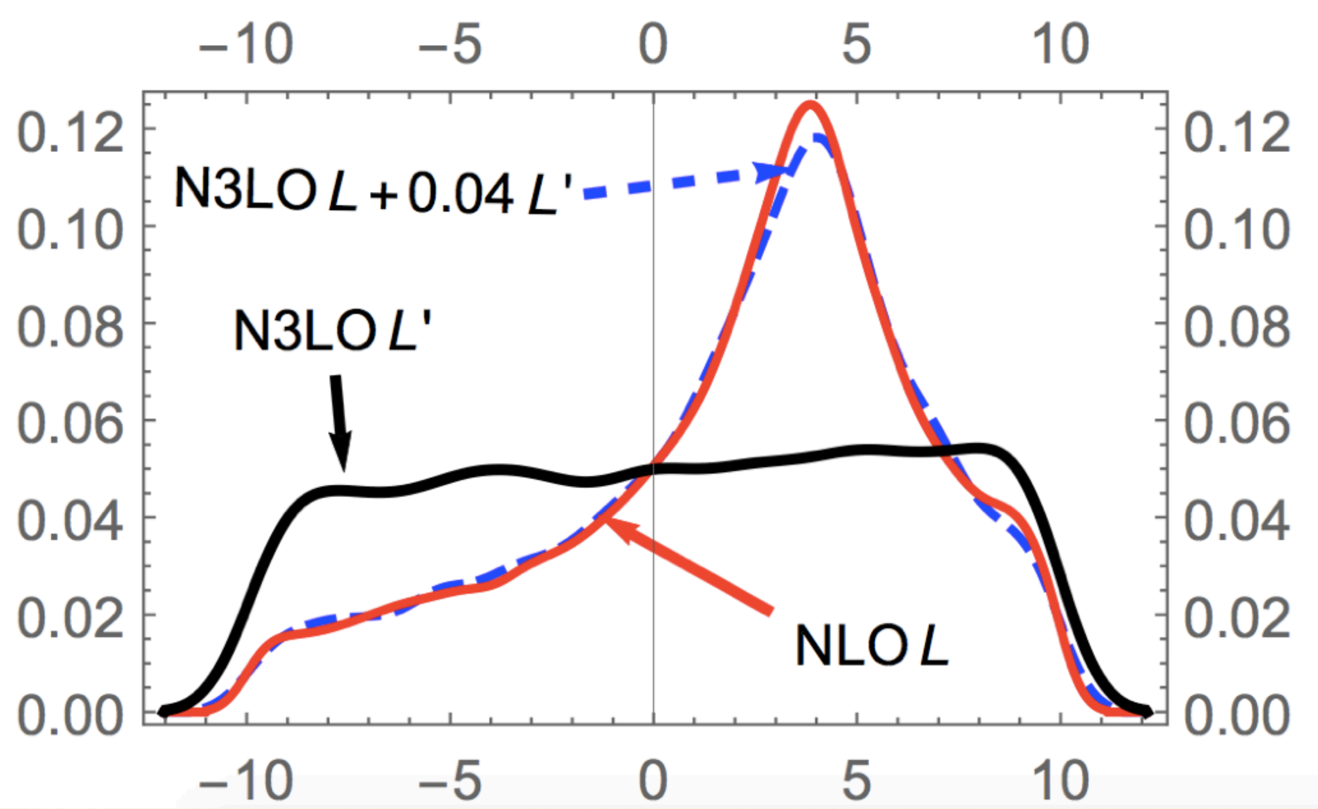
- Is it a "good fit"?
- Did the experimentalists understand their systematic errors?
- Are there parameters that are not well constrained by these data?



Truncation error

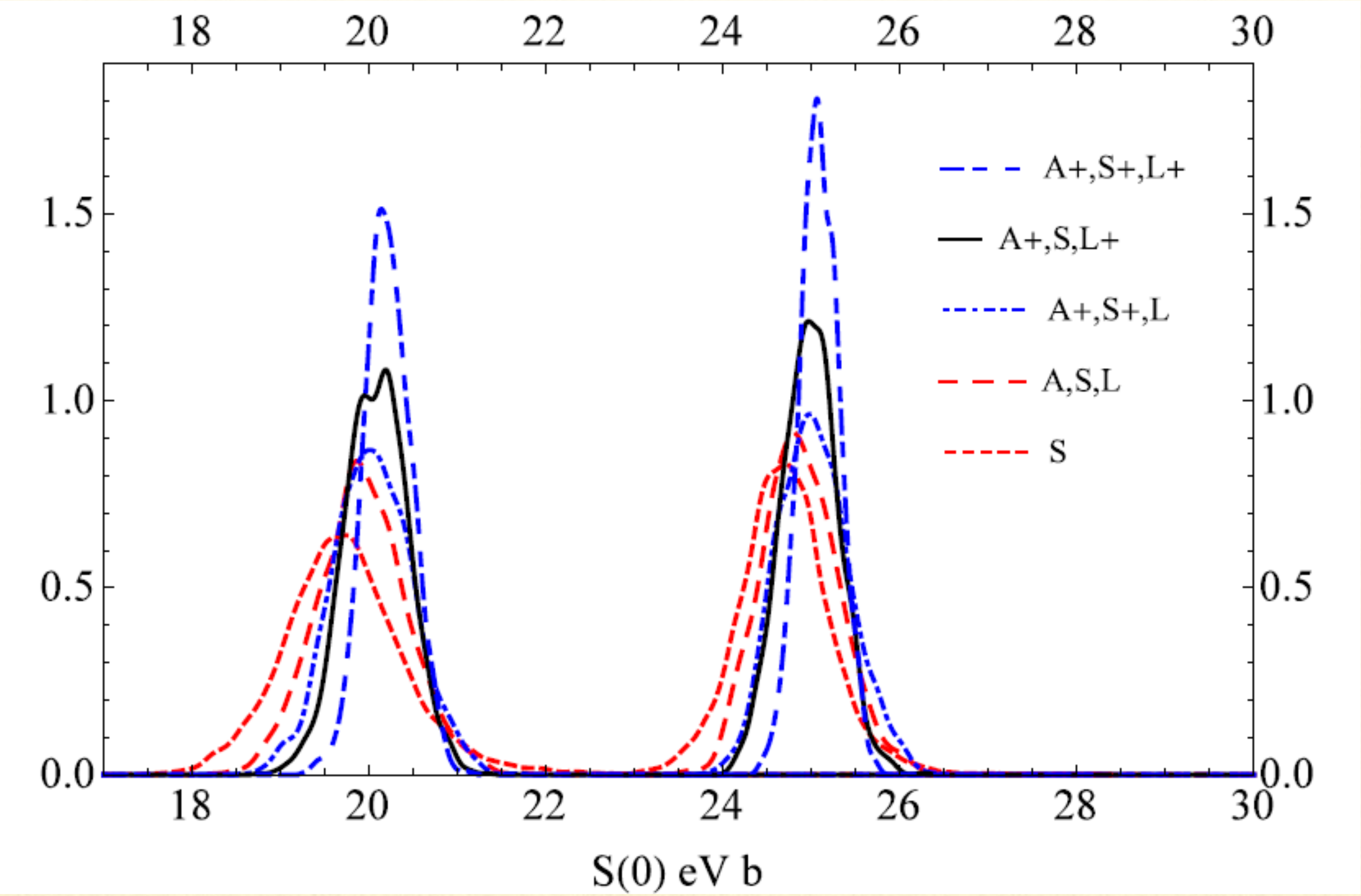
- N2LO correction=0 (technically only in absence of excited state)
- EFT s-wave scattering corrections (shape parameter)~0.8%
- E2, M1 contributions < 0.01%, Radiative corrections: ~0.1%
- So first correction is at N3LO, i.e., $\overline{L}_i \to \overline{L}_i + k^2 \overline{L}_i'$





Planning improvements

Use extrapolant to simulate impact of hypothetical future data that could inform posterior pdf for S(0)



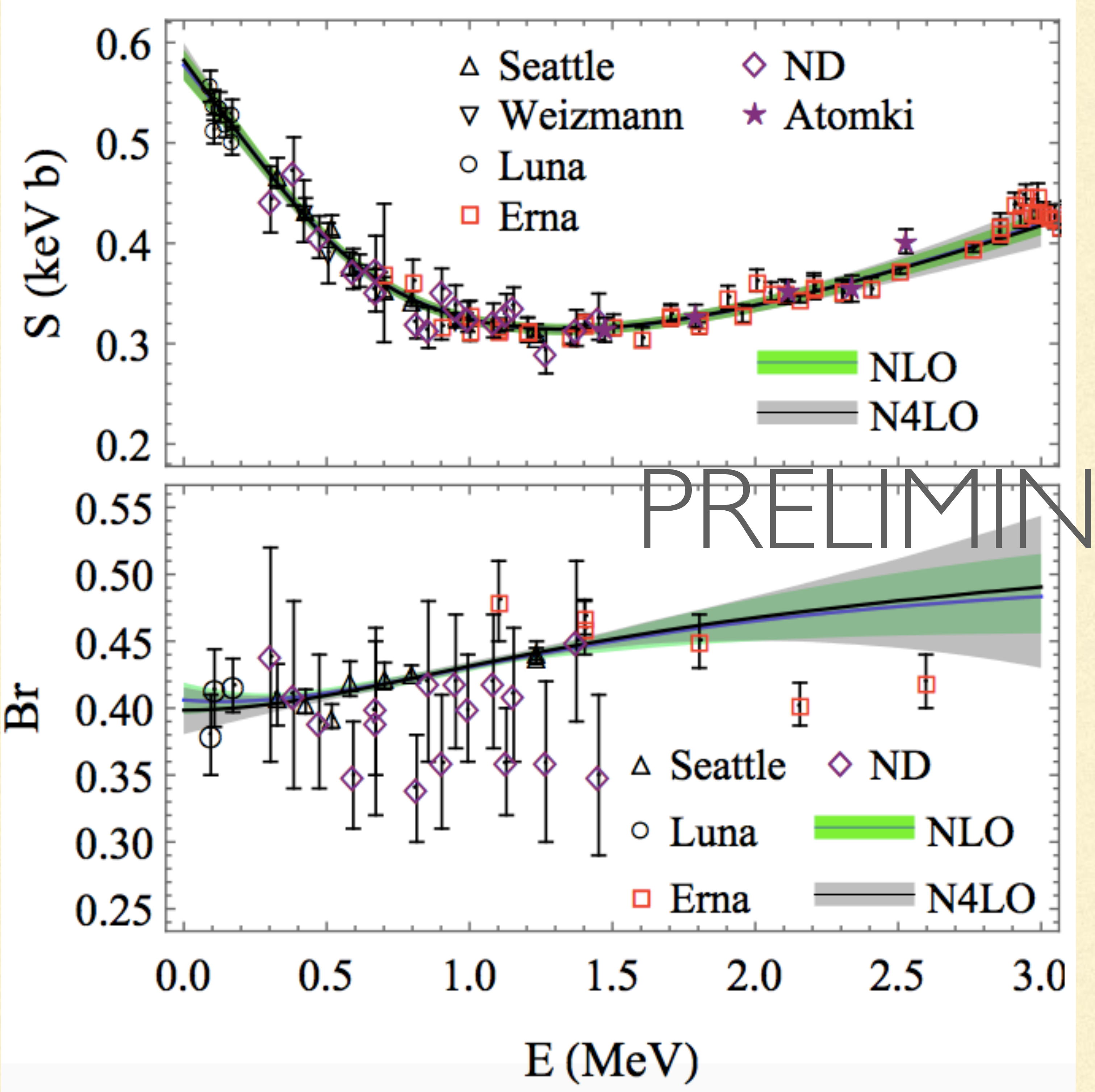
Left-to-right:
42 data points all of similar quality to Junghans et al.

A: ANC
S: $a_{S=1}$ and $a_{S=2}$
L: short-distance

Note that 1 keV uncertainty in S_{1p} of ^{8}B may not be negligible effect

A sneak peek at ^{3}He(^{4}He,ɣ)

Zhang, Nollett, DP, in preparation



PRELIMINARY

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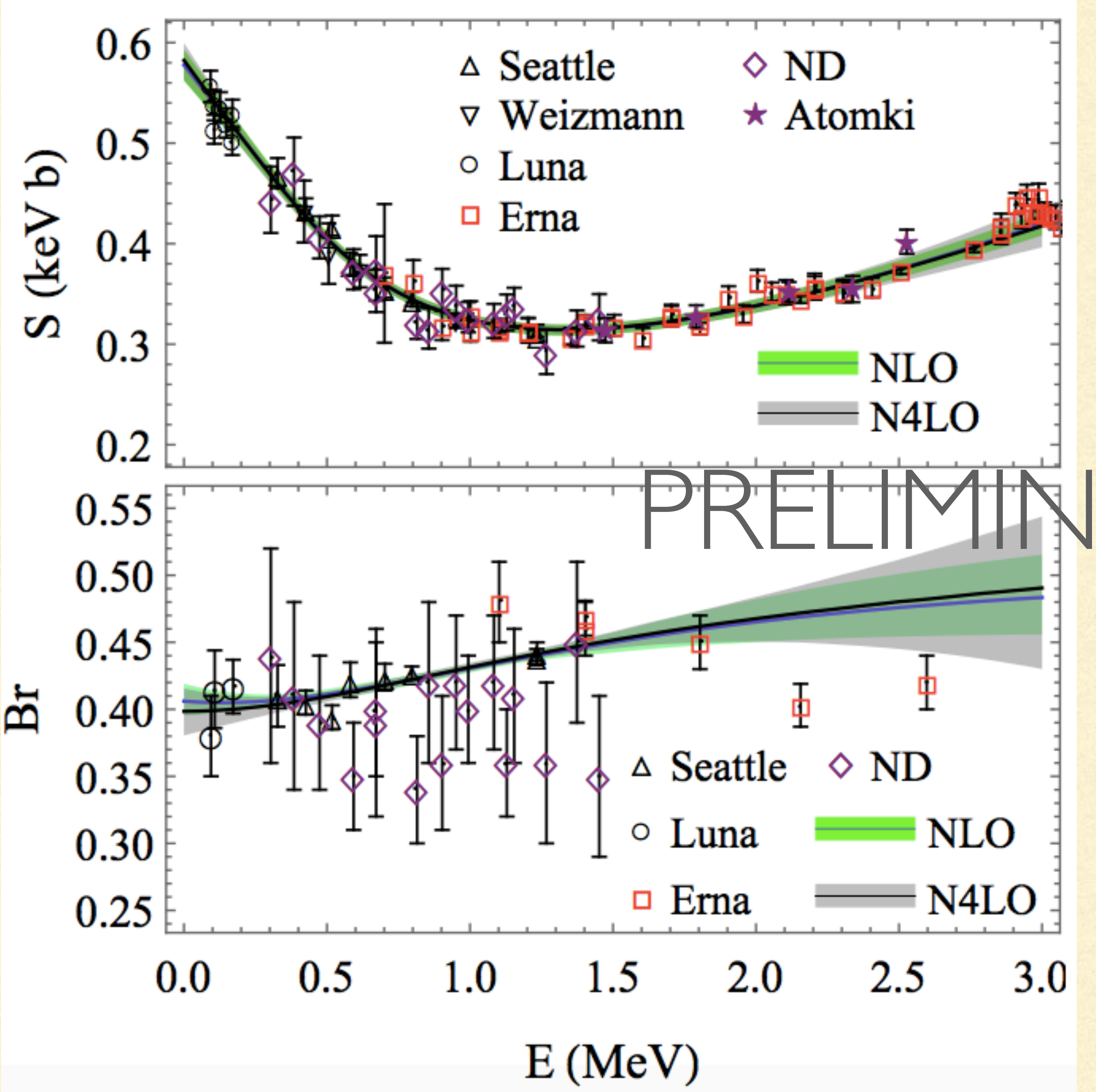
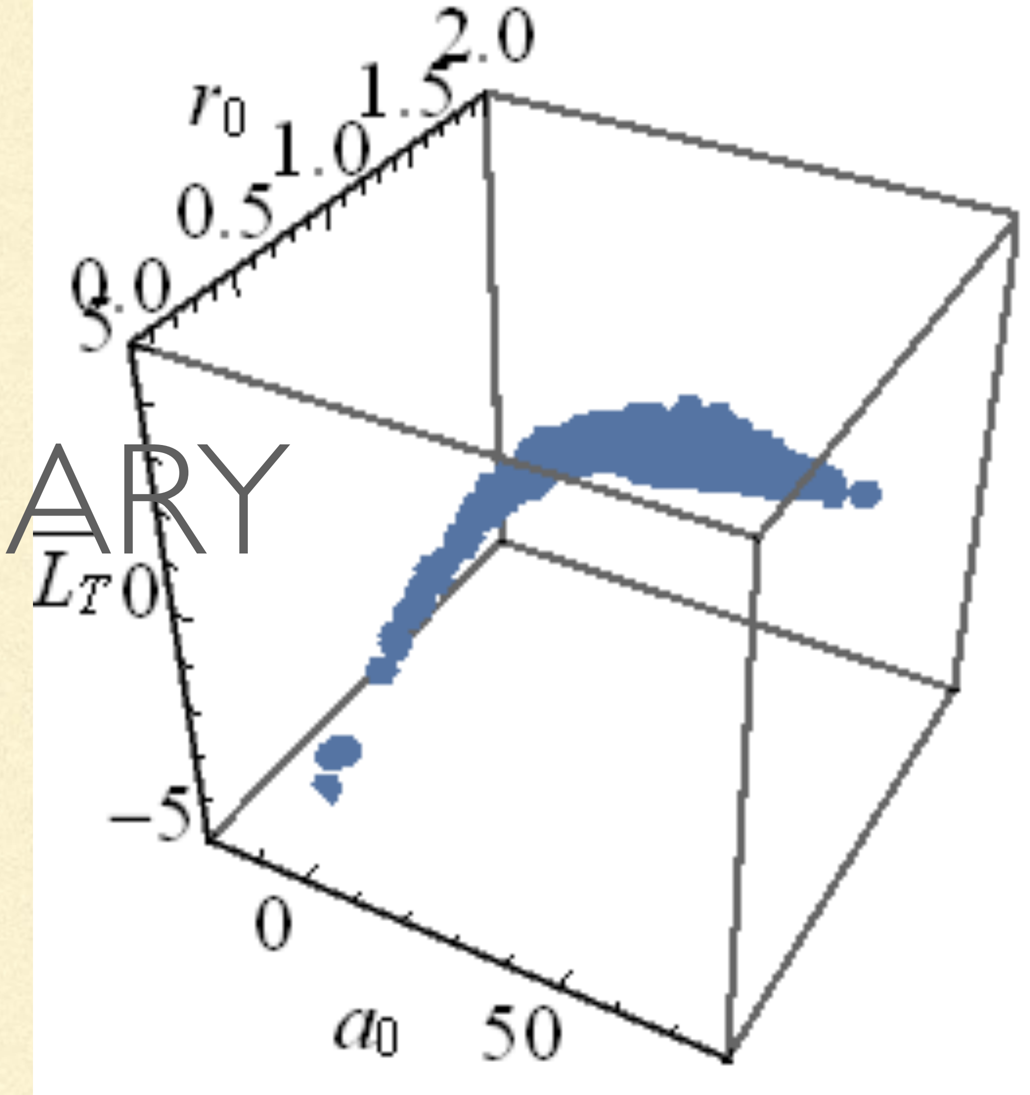
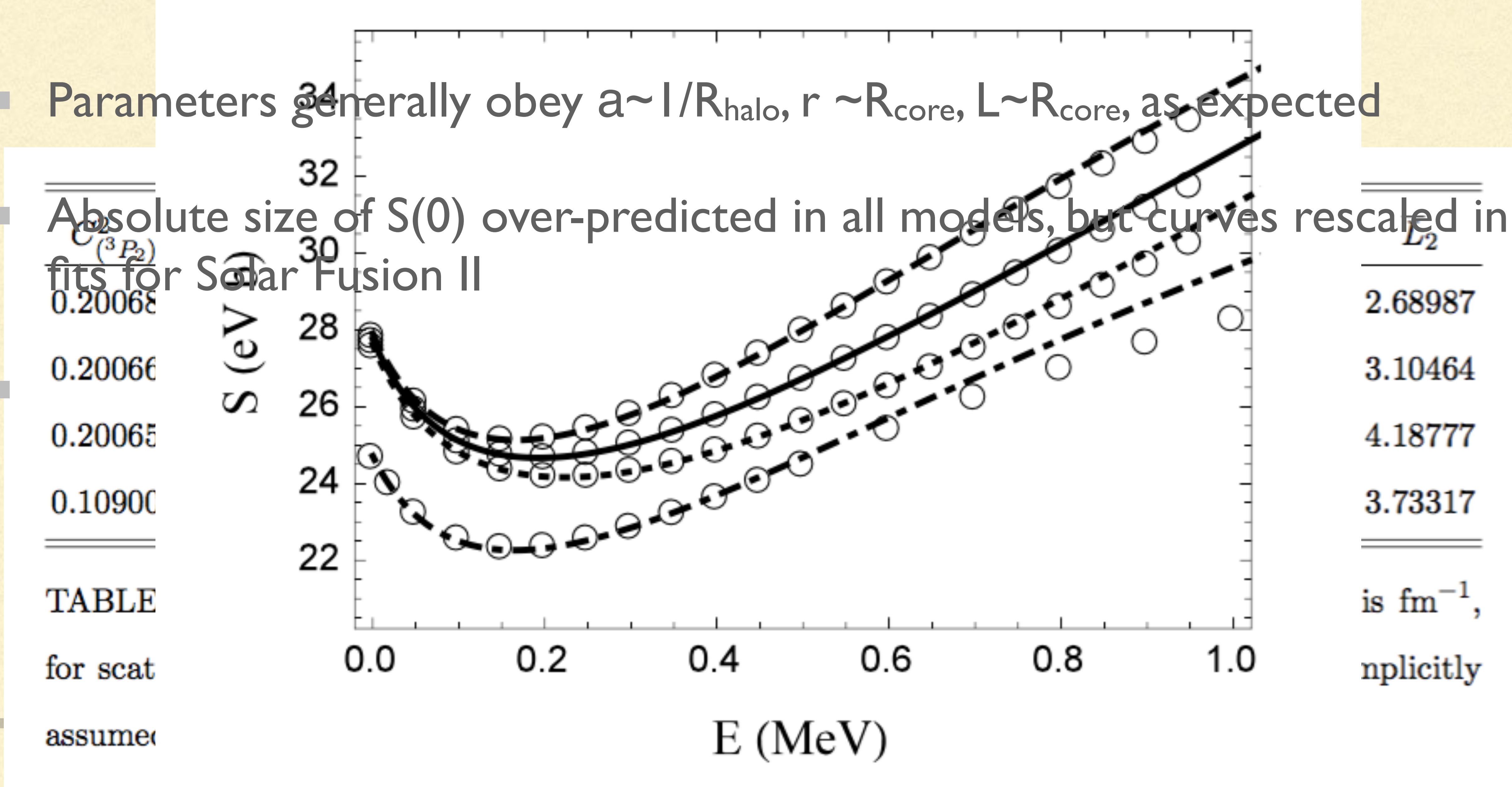




PRELIMINARY

Halo EFT as a “super model”

- Halo EFT is also the EFT of all the models used to extrapolate the cross section in “Solar Fusion II”
- Differences are sub-% level between 0 and 0.5 MeV
- Parameters generally obey a~1/R$_{\text{halo}}$, r ~R$_{\text{core}}$, L~R$_{\text{core}}$, as expected
- Absolute size of S(0) over-predicted in all models, but curves rescaled in fits for Solar Fusion II



$C^2_{(^3P_2)}$	L_2
0.20068	2.68987
0.20066	3.10464
0.20065	4.18777
0.10900	3.73317

TABLE … is fm^{-1},

for scat … nplicitly

assume…